PART- A

QA1. Solve the following nonlinear differential equation by transforming into a linear equation:

$$2(\mu+1)\frac{du}{dx}-\frac{2}{x}(\mu+1)^2=x^4,\ \mu(1)=\sqrt{\frac{2}{3}}-1$$

QA2.

QA3. A particle of mass m is constrained to slide under gravity on a smooth wire in the shape of vertical circle with radius R. The wire is rotating about the vertical diameter with constant angular velocity Ω

(i) Set up the Lagrangian and write the equations of motions . Write the answers in terms of the angular displacement of the particle from the downward vertical with the center of the circle as the origin

(ii) Derive the Hamiltonian of the system. Is it conserved?

QA4. Obtain the following virial expansion for the ideal Bose gas of identical particles of mass m each, at temperature T

$$\frac{Pv}{k_BT}=1-\frac{\bar{n}}{4\sqrt{2}}+\left(\frac{1}{8}-\frac{2}{9\sqrt{3}}\right)\bar{n}^2+\ldots$$

in power of $n\wedge^3=\bar{n}$, where n is the average number of particles per unit volume of the gas and $\wedge=h/\sqrt{2\pi mk_BT}$ is the thermal wavelength. Symbols have their usual meaning.

QA5. A circular disc of radius R has a uniform change density σ. It is revolving about an axis passing perpendicularly through its center with angular velocity ω. Find the value of magnetic induction at the center.

PART - B

QB1. Consider the 4×4 matrix

$$A=\begin{pmatrix}1&1&1&1\\1&1&0&0\\1&0&1&0\\1&0&0&1\end{pmatrix}$$

Find the eigenvalues of $e^{\alpha A}$ where α is a real number. Are any eigenvalues degenerate?

QB2. An electron is confined inside a cubic box of side $10\ \overset{o}{A}$. Calculate the force exerted on one of the faces of the box if the electron is in the ground state.

QB3. A particle of unit mass moves in the $x-y$ plane under the potential

$$V(x,y)=\frac{k}{2}\left(x^2+y^2+2\alpha xy\right), -1<\alpha<1$$

where k is a constant. Find the frequencies of normal modes of oscillations

QB4. Consider a mono-atomic gas consisting of particles of mass m at temperature T. Let $f(v)dv$ denote the probability that a particle of the gas will have speed between v and $v+dv$. Write down the expression of $f(v)$ according to the Maxwell's velocity distribution law. Calculate $\langle v^4 \rangle$.

QB5. An amplifier has an ideal gain of 100 which changes by 20% due to charge in internal parameters. After a feedback circuit is incorporated this fluctuation is limited to 2%. Find the feedback parameter of the circuit

QB6. the rare-earth metal Dysprosium (Dy) has a saturation magnetization of $2.4\times10^6\ A/m$. Its density is $8.5\times10^3\ kg/m^3$ and molar atomic mass is $162.5\times10^{-3}\ kg/mole$. Find the magnetic moment per atom in terms of effective number of Bohr magneton (μ_B)

QB7. An element has a cubic lattice with unit cell dimension a. It contributes one valence electron per atom for elecrtrical conduction. Using the free electron model, calculate the Fermi wave vector of conduction electrons and compare it with the first Brillouin zone dimension.

QB8. Show the energy-level diagram for Zeeman splitting of the cadmium red line $(\lambda = 643.8\ nm)$ for the transition between spectroscopic levels ${}^1D_2 \rightarrow {}^1P_1$.

QB9. Calculate the speed at which the total energy (E_{total}) of a free particle equals twice its rest mass energy.

QB10. A dielectric fiber of a transparent material can be used to guide electromagnetic waves under the condition of total internal reflection. Determine the minimum dielectric constant of the medium so that a wave incident at an angle on one end will be confined within the fiber until it emerges from the other end.