Pre Ph.D. / Ph.D. Entrance 2014

PART – A

Note: Answer all questions. Each question carries 6 marks.

Q1. Using the method of residues, calculate the integral $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$

Q2. A particle of mass m moves in a one-dimensional box located between $x=-\frac{L}{2}$ and $x=\frac{L}{2}$. Thus the potential function $V(x)=0$ if $|x|<\frac{L}{2}$ but is infinite otherwise. Suppose, at $t=0$, the wave function of the particle inside the box is given by $\psi(x)=\beta\left(\frac{L^2}{4}-x^2\right)$, where β is a suitable normalization factor.

(a) Calculate the value of β.

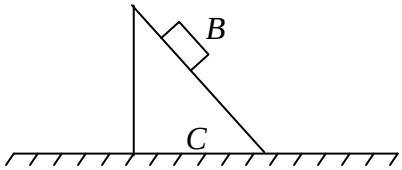
(b) Calculate the wave function of the particle for $t>0$.

Q3. A block B of mass m is lying on the frictionless top surface of a triangular block C of mass M as shown in the figure below. The flat lower surface of C is resting on a frictionless horizontal surface.

(a) Write down the Lagrangian of this system of two masses.

(b) Derive the Lagrange's equation(s) of motion.

(c) Suppose both B and C are held at rest initially. Then, at $t=0$, we release them so that B can slide freely down the surface of C under the action of gravity and C is also free to recoil. Solve the Lagrange's equation(s) of motion to find how the vertical coordinate of the block B will change with time.



Q4. Consider a spherical shell with inner radius a and outer radius b. Throughout this shell, there is electrostatic charge with uniform charge density ρ.

(a) Calculate the electric field for every value of r, the radial coordinate.

(b) Calculate the electrostatic potential for $r<a$.

Q5. Consider a one-dimensional quantum harmonic oscillator of frequency v in equilibrium with a heat bath at temperature T. Calculate the partition function, average energy and root mean square fluctuation of energy for this system.

PART – B

Note: Answer all questions. Each question carries 4 marks.

Q1. For dimensional analysis, we usually take mass, length and time as the basic physical variables with dimensions $[M]$, $[L]$ and $[T]$. Suppose, deviating from this normal practice, we choose to use speed, angular momentum and frequency as the basic variables and denote their dimensions by $[E]$, $[P]$ and $[H]$. What would be the dimensions of linear momentum and torque in terms of $[E]$, $[P]$ and $[H]$?

Q2. A radioactive element A decays to B with the decay rate λ_A. In turn, B decays to C with the decay rate λ_B. Suppose, initially, only A-type nuclei are present in such a radioactive sample. After what time would the number of B-type nuclei in this sample reach its maximum value?

Q3. For a free relativistic particle of rest mass m the Hamiltonian is $H = \left(\vec{p}.\vec{p}c^2 + m^2c^4\right)^{\frac{1}{2}}$. Here $\vec{p}$ is the momentum vector and c is the speed of light in vacuum. Derive the expression of the Lagrangian for this particle.

Q4. Evaluate the integral $\int_0^\infty dx e^{-x^2} \int_0^x dy e^{-y^2}$.

Q5. A spin-$\frac{1}{2}$ particle is in the spin state

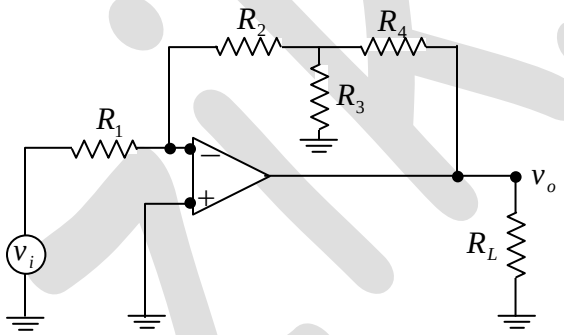
$$|\psi\rangle = \cos\left(\frac{\theta}{2}\right)e^{-i\varphi}\left|\frac{1}{2},\ \frac{1}{2}\right\rangle + \sin\left(\frac{\theta}{2}\right)\left|\frac{1}{2},-\frac{1}{2}\right\rangle$$

$\left|\frac{1}{2},\frac{1}{2}\right\rangle$ and $\left|\frac{1}{2},-\frac{1}{2}\right\rangle$ are the eigenstates of S_z, the operator for the z-component of spin angular momentum, with eigenvalues of $\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$, respectively. Calculate the expectation values of S_x and S_y in the state $|\psi\rangle$.

Q6. A metallic block of mass $2\,kg$ and specific heat $2.5\,cal/K$ per gram is initially at a temperature of $80^{\circ}C$. It is then dropped into a swimming pool, the temperature of this pool being $25^{\circ}C$. How much will the entropy of this combined system of the metallic block and the swimming pool change by the time thermal equilibrium is established? Does the entropy increase or decrease?

Q7. Consider a divalent metallic element in a crystalline solid state with a simple cubic primitive cell of side 4 angstrom. In the free electron approximation, what is the length of the Fermi wave vector for this metal?

Q8. In the following inverting feedback circuit of an operation amplifier, calculate the voltage gain. Take $R_1 = R_2 = R_4 = 100k\,\Omega$, $R_3 = R_L = 10k\,\Omega$.



Q9. Calculate the binding energy of a positronium, which is a bound state formed by an electron and its anti-particle (positron). [You may use any results derived in standard introductory textbooks on Quantum Mechanics]

Q10. Consider an electron moving with a kinetic energy of $20\,MeV$. By what percentage is its speed different from c (the speed of light in vacuum)?