JNU-ENTRANCE EXAMINATION, 2009

Ph.d (Physical Science)

Maximum Marks: 70

PART-A

NOTE: Answer **all** questions. Each question carries 6 marks.

Q.1 Find the solution of the differential equation

$$\frac{dy}{dx} + y(x)\tan x = \cos^2 x$$

given that $y(0) = 13$.

Q.2 A particle is moving on a circle of radius 1 *cm*. At a particular instant of time, its *angular velocity* is 2 radians/s and the rate of increase of angular velocity is 3 radians/s^2. Calculate the magnitude of the *linear acceleration* vector at this moment.

Q.3 The earth (6×10^{24} kg) rotates about the sun (2×10^{30} kg) in an approximately circular orbit of radius 1.5×10^{11} m. Apply Bohr's quantum theory to this problem to find the principal quantum number n corresponding to the earth's orbit.

Q.4 Consider a grounded conducting plane that is horizontal and is infinite in extent. A point charge Q is at a height h above this plane. Calculate the total induced charge on the conducting plane within a circle of radius R, the centre of this circle being directly below the point charge Q.

Q.5 Consider a one-dimensional anharmonic oscillator in contact with a heat bath at temperature T. The Hamiltonian of this oscillator is given by

$$H = \frac{1}{2m}p^2 + bx^{2n}$$

where n (> 1) is an integer and b is ral and positive.

(a) Show that the temperature dependence of the classical canonical partition function is of the form T^{α}, where α depends on n only. [You do not need to evaluate any integral]

(b) How does the average energy depend on temperature?

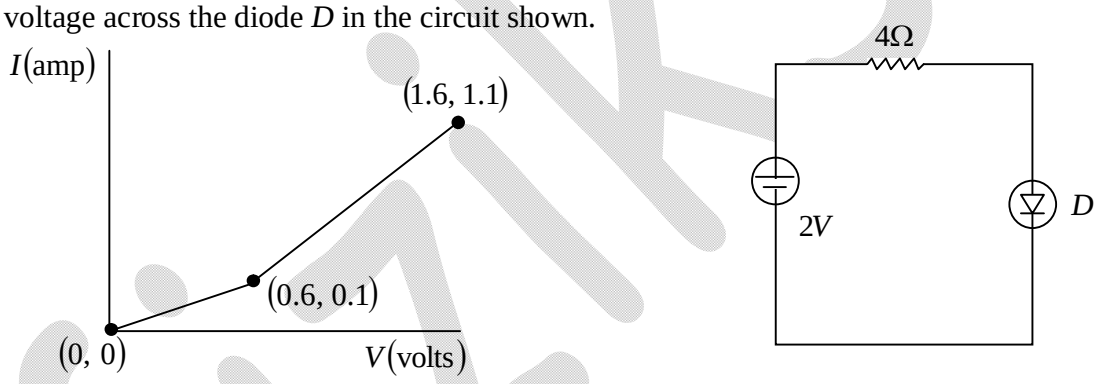
PART – B

NOTE: Answer **all** questions. Each question carries 4 marks.

Q.1 A and B are n x n matrices such that $A^2 = 1$, $B^2 = 1$ (where 1 is the n x n identify matrix) and $AB = -BA$. Calculate the trace of the matrix A.

Q.2 Two identical point charges, each of strength Q, are located at the same height h above a horizontally placed infinite plate that is perfectly conducting. The separation between the two charges is D. Calculate f_h and f_v, where f_h and f_v are the magnitudes of the horizontal and the vertical components respectively of the electrostatic force on either of the two charges.

Q.3 The piecewise linear I-V characteristic of a diode D is shown below. Find the current and the voltage across the diode D in the circuit shown.

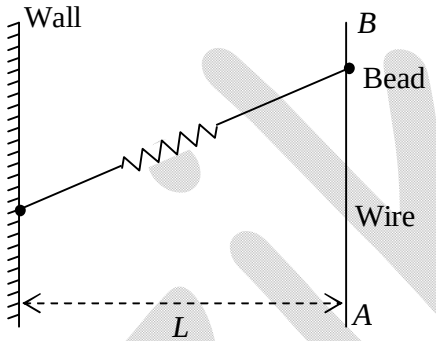


Q.4 In an apparatus ions of a particular type are created at a source. These are then accelerated from rest through a fixed potential drop V in the positive z-direction. After this the ions enter a region in which there is a uniform electric field along the positive x-direction and a uniform magnetic field in the positive y-direction. Find the condition that has to be satisfied so that these ions will pass through this region without any deflection.

Q.5 The planes of two linear polarizer plates P_1 and P_2 are parallel to each other. The plate P_1 is fixed, but P_2 is rotating with an angular frequency ω about the common normal to the plates. An initially unpolarized beam of light falls perpendicularly on P_1 and then passes through P_2 before it is detected. How will the intensity of the beam emerging from P_2 vary with time?

Q.6 A quantum particle of mass m is moving in a one-dimensional box of length L. Compute the expectation values of the momentum operator and the kinetic energy operator when the particle is in the ground state.

Q.7 Consider a spring of equilibrium length l. It is fixed at one end to a rigid wall. The spring is now stretched and its other end is attached to a bead of mass m that is constrained to move without friction along a vertical wire AB (figure). The wire is at a distance L from the wall. Write the Lagrangian of the bead and derive the equation(s) of motion. (Ignore the effects of gravity)



Q.8 Consider an ideal gas in equilibrium at temperature T. If a measurement were to be made of the x-component of velocity for all the gas particles at a particular instant, what fraction of these values will be in the range from v_1 to v_2? (There is no need to evaluate the integrals).

Q.9 Consider a system of four non-interacting, distinguishable particles. Each particle can occupy one or two possible quantum states with energy $\pm E$.

(a) How many quantum states are possible for this system with total energy equal to zero?

(b) Repeat the calculation with total energy equal to $2E$.

Q.10 The mean lifetime of muons in their rest frame is 2.2×10^{-6} s. What is the average distance traveled by muons moving with a speed of $0.6\,c$ (c is the speed of light in vacuum).