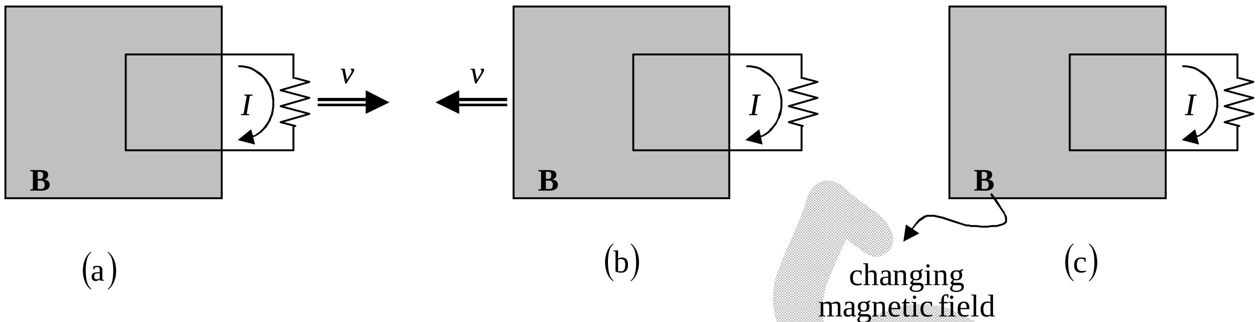
Chapter - 4

Electromagnetic Induction

4.1 Faraday's Law



Experiment 1: He pulled a loop of wire to the right through a magnetic field. A current flowed in the loop (Figure a).

Experiment 2: He moved the magnet to the left, holding the loop still. Again, a current flowed in the loop (Figure b).

Experiment 3: With both the loop and the magnet at rest, he changed the strength of the field (he used an electromagnet, and varied the current in the coil). Once again current flowed in the loop (Figure c).

Thus, universal flux rule is that, whenever (and for whatever reason) the magnetic flux through a loop changes, an e.m.f. (ε) will appear in the loop $\varepsilon = -\frac{d\Phi}{dt}$

In experiment 2, *A changing magnetic field induces an electric field.*

It is this "induced" electric field that accounts for the e.m.f.

Also the induced e.m.f $\varepsilon = \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$ (where magnetic flux $\Phi = \int \vec{B}.d\vec{a}$)

Then $\vec{E}$ is related to the change in $\vec{B}$ by the equation

$$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}.d\vec{a} \qquad \Rightarrow \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

4.1.1 Lenz's Law

In Faraday's law negative sign represents the **Lenz's law.** (The induced current will flow in such a direction that the flux it produces tends to cancel the change).

For example if the magnetic flux is increasing then induced e.m.f will try to reduce and vice versa.

Example: A long solenoid, of radius a, is driven by alternating current, so that the field inside is sinusoidal $\vec{B}(t) = B_0 \cos(\omega t)\hat{z}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

Solution: Magnetic flux through the loop $\phi = \vec{B}.\vec{A} = B_0 \cos(\omega t) \times \frac{\pi a^2}{4} = \frac{1}{4}\pi a^2 B_0 \cos(\omega t)$

Induced emf $\varepsilon(t) = -\frac{d\phi}{dt} = \frac{1}{4}\pi a^2 B_0 \omega \sin\omega t = \varepsilon_0 \sin\omega t \Rightarrow \varepsilon_0 = \frac{1}{4}\pi a^2 B_0 \omega$

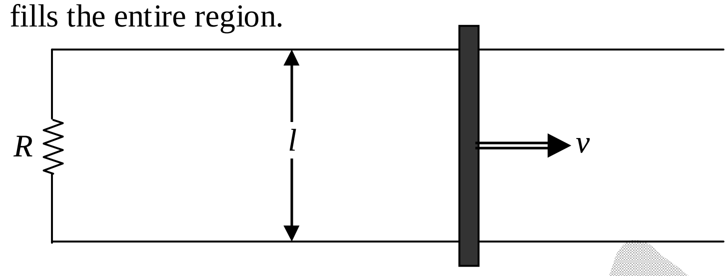
Induced current $i(t) = \frac{\varepsilon(t)}{R} = \frac{\pi a^2 B_0 \omega \sin\omega t}{4R} = i_0 \sin\omega t \Rightarrow i_0 = \frac{\pi a^2 B_0 \omega}{4R}$

Example: A square loop (side a) is mounted on a vertical shaft and rotated at angular velocity ω. A uniform magnetic field $\vec{B}$ points to the right. Find the induced emf $\varepsilon(t)$ for this alternating current generator.

Solution: Magnetic flux $\phi = \vec{B}.\vec{A} = BA\cos\theta = Ba^2 \cos\omega t$

Induced emf $\varepsilon(t) = -\frac{d\phi}{dt} = Ba^2 \omega \sin\omega t$

Example: A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart. A resistor R is connected across the rails and a uniform magnetic field $\vec{B}$, pointing into the page, fills the entire region.



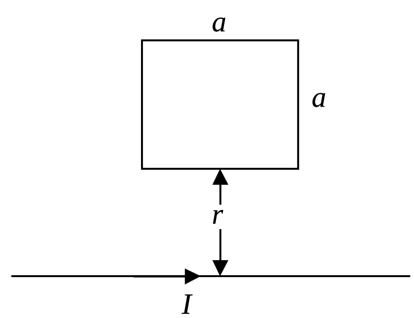
(a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow?

(b) What is the magnetic force on the bar?

Solution: (a) $\varepsilon = -\dfrac{d\Phi}{dt} = -Bl\dfrac{dx}{dt} = -Blv$; $\varepsilon = IR \Rightarrow I = \dfrac{Blv}{R}$ (downward in R).

(b) $F = IlB = \dfrac{B^2 l^2 v}{R}$ (to the left)

Example: A square loop of wire (side a) lies on a table, a distance r from a very long straight wire, which carries a current I.



(a) Find the flux of $\vec{B}$ through the loop.

(b) If some one now pulls the loop directly away from the wire, at speed v, what emf is generated? In what direction does the current flow?

(c) What if the loop is pulled to the right at speed v, instead of moving away?

Solution: (a) $\Phi = \int \vec{B}.d\vec{a} = \dfrac{\mu_0 I}{2\pi}\displaystyle\int_r^{r+a} \dfrac{1}{r}(adr) = \dfrac{\mu_0 I}{2\pi}\ln\left(\dfrac{r+a}{r}\right)$

(b) $\varepsilon = -\dfrac{d\Phi}{dt} = -\dfrac{\mu_0 Ia}{2\pi}\dfrac{d}{dt}\ln\left(\dfrac{r+a}{r}\right)$

$$\Rightarrow \varepsilon = -\frac{\mu_0 Ia}{2\pi}\left(\frac{1}{r+a}\frac{dr}{dt} - \frac{1}{r}\frac{dr}{dt}\right) = \frac{\mu_0 Ia^2 v}{2\pi r(r+a)} (counter\ clockwise)$$

(c) Flux is constant so $\varepsilon = 0$.

Example: A long solenoid, of radius *a* and *n* turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\phi}$ direction. Find the electric field (magnitude and direction) at a distance *r* from the axis (both inside and outside the solenoid).

Solution: Field due to solenoid $\vec{B}(t) = \mu_0 n I(t)\hat{z}$ *inside* and zero outside.

Inside solenoid ($r < a$):

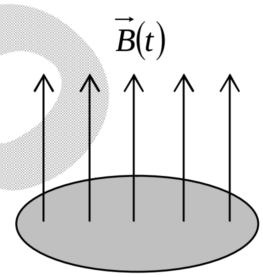
$$\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int \vec{B}.d\vec{a} \Rightarrow \left|\vec{E}\right|(2\pi r) = -\mu_0 n \frac{dI(t)}{dt}\times \pi r^2 \Rightarrow \vec{E} = -\mu_0 n \frac{r}{2}\frac{dI(t)}{dt}\hat{\phi}.$$

Outside solenoid ($r > a$):

$$\oint \vec{E}.d\vec{l} = -\frac{d}{dt}\int \vec{B}.d\vec{a} \Rightarrow \left|\vec{E}\right|(2\pi r) = -\mu_0 n \frac{dI(t)}{dt}\times \pi a^2$$

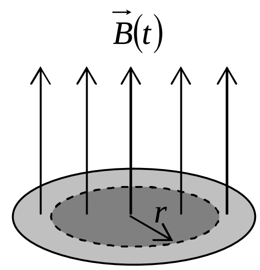
$$\Rightarrow \vec{E} = -\frac{\mu_0 n a^2}{2r}\frac{dI(t)}{dt}\hat{\phi}.$$

Example: A uniform magnetic field $\vec{B}(t)$, pointing straight up, fills the shaded circular region of figure shown below. If $\vec{B}$ is changing with time, what is the induced electric field?



Solution: $\vec{E}(t)$ points in the circumferential direction, just like the magnetic field inside a long straight wire carrying a uniform current density.

Draw an Amperian loop of radius *r* and apply Faraday's Law:



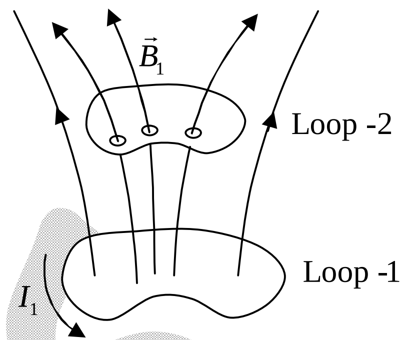
$$\oint \vec{E}.d\vec{l} = -\frac{d\phi}{dt} \Rightarrow \left|\vec{E}\right|(2\pi r) = -\frac{d}{dt}\left(\pi r^2 B(t)\right) = -\pi r^2\frac{dB(t)}{dt}.$$

Thus $$\vec{E} = -\frac{r}{2}\frac{dB(t)}{dt}\hat{\phi}.$$

If $\vec{B}$ is increasing, $\vec{E}$ runs clockwise, as viewed from above.

4.1.2 Inductance

If a steady current I_1 flows in a loop 1, it produces magnetic field $\vec{B}_1$. Some of the field lines pass through loop 2, let Φ_2 be the flux of $\vec{B}_1$ through 2.



From Biot-Savart law, $\vec{B}_1 = \frac{\mu_0}{4\pi} I_1 \oint \frac{d\vec{l}_1 \times \hat{R}}{R^2}$,

Therefore flux through loop 2 is $\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2$.

Thus $\Phi_2 = M_{21} I_1$, where M_{21} is the constant of proportionality; it is known as the **mutual inductance** of the two loops. Now

$$\Phi_2 = \int \vec{B}_1 \cdot d\vec{a}_2 = \int \left(\vec{\nabla} \times \vec{A}_1\right) . d\vec{a}_2 = \oint \vec{A}_1 . d\vec{l}_2$$

Since $\vec{A}_1 = \frac{\mu_0 I_1}{4\pi} \oint \frac{d\vec{l}_1}{R} \Rightarrow \Phi_2 = \frac{\mu_0 I_1}{4\pi} \oint \left(\oint \frac{d\vec{l}_1}{R} \right) . d\vec{l}_2$

$$\Phi_2 = M_{21} I_1 \Rightarrow M_{21} = \frac{\mu_0 I_1}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{R}$$

This is the Neumann formula; it involves double line integral-one integration around loop1, the other around loop2.

Thus

(a) M_{21} is a purely geometrical quantity depends on sizes, shapes and relative position of two loops.

(b) $M_{21} = M_{12} = M$

If flux through loop 2, varies then induce emf in loop 2 is

$$\varepsilon_2 = -\frac{d\phi_2}{dt} = -M \frac{dI_1}{dt}.$$

Changing current not only induces an emf in any nearby loops, it also induces an emf in the source loop itself. Again field (and therefore flux) is proportional to the current.

$\Phi = LI$ where L is self inductance of the loop

If the current changes, the emf induced in the same loop is

$$\varepsilon = -L\frac{dI}{dt}.$$

Inductance is measured in **henries** (H); a henry is a volt-second per ampere.

Inductance (like capacitance) is an intrinsically positive quantity. Lenz's law, which is enforced by minus sign, which means the emf is in such a direction to oppose and change in current. For this reason, it is called a **back emf.** Whenever we try to alter the current, we must fight against this back emf.

4.1.3 Energy Stored in the field

It takes a certain amount of energy to start a current flowing in a circuit. The work done on a unit charge, against the back emf, in one trip around the circuit is $-\varepsilon$ (the mines sign is due to the fact that work is being done by us against the emf, not the work done by the emf). The amount of charge per unit time passing down the wire is I. So the total work done per unit time is, $\frac{dW}{dt} = -\varepsilon I = LI\frac{dI}{dt}$

If we start with zero current and build it up to a final value I,

The work done (Integrating the last equation over time) is $W = \frac{1}{2}LI^2$

Since $\Phi = LI = \int_S \vec{B}.d\vec{a} = \int_S \left(\vec{\nabla}\times\vec{A}\right).d\vec{a} = \oint_P \vec{A}.d\vec{l}$, where P is the perimeter of the loop and S is any surface bounded by P.

Therefore $\quad W = \frac{1}{2\mu_0} I\oint \vec{A}.d\vec{l} \Rightarrow W = \frac{1}{2\mu_0}\oint\left(\vec{A}.\vec{I}\right)dl$

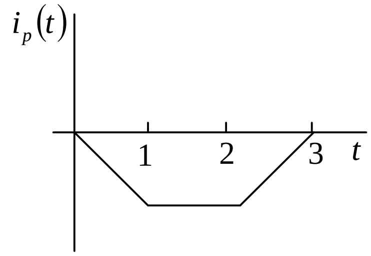
The generalization to volume current is: $W = \frac{1}{2}\int_V \left(\vec{A}.\vec{J}\right)d\tau$

We can simplify above equation as

$$W = \frac{1}{2\mu_0}\int_{all\ space} B^2 d\tau$$

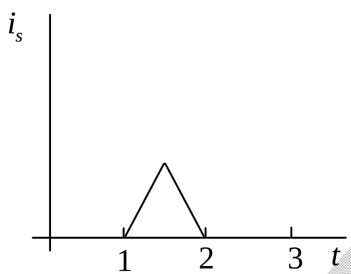
MCQ (Multiple Choice Questions)

Q1. A current i_p flows through the primary coil of a transformer. The graph of $i_p(t)$ as a function of time t is shown in the figure below.

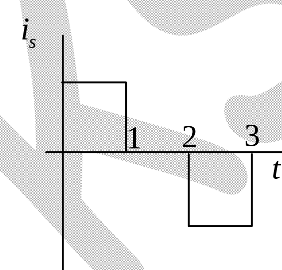


Which of the following graphs represents the current i_S in the secondary coil?

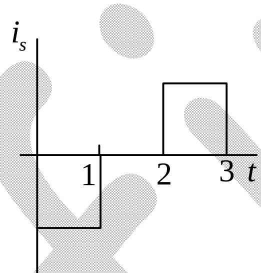
(a)



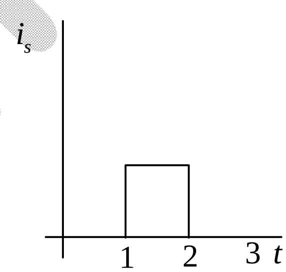
(b)



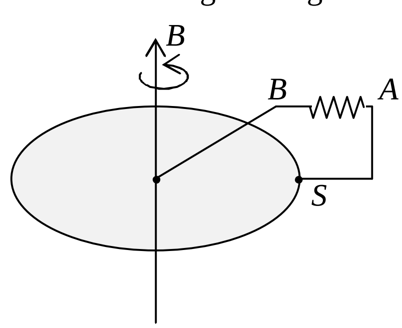
(c)



(d)



Q2. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field pointing up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contact. The current that flows through the resistor is



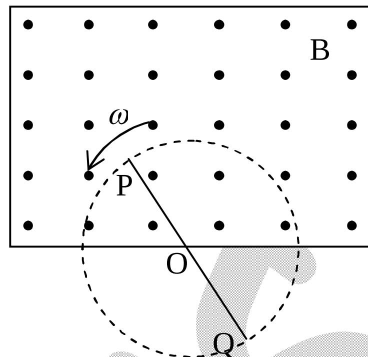
(a) zero

(b) DC from A to B

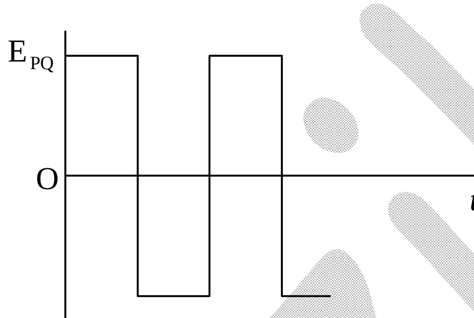
(c) DC from B to A

(d) AC

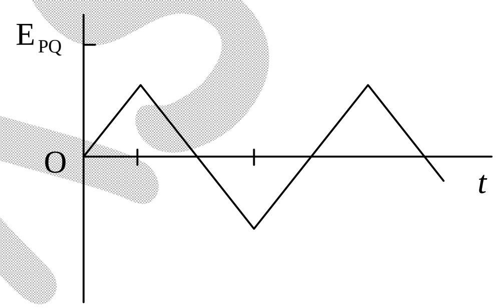
Q3. A uniform and constant magnetic field B coming out of the plane of the paper exists in a rectangular region as shown in the figure. A conducting rod PQ is rotated about O with a uniform angular speed ω in the plane of the paper. The emf E_{PQ} induced between P and Q is best represented by the graph



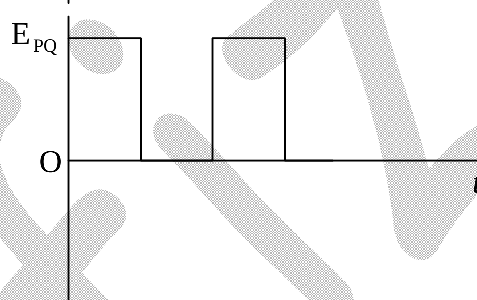
(a)



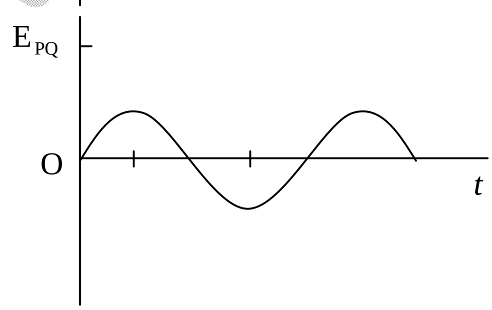
(b)



(c)



(d)



Q4. Consider a solenoid of radius R with n turns per unit length, in which a time dependent current $I = I_0 \sin \omega t$ (where $\omega R / c << 1$) flows. The magnitude of the electric field at a perpendicular distance $r > R$ from the axis of symmetry of the solenoid, is

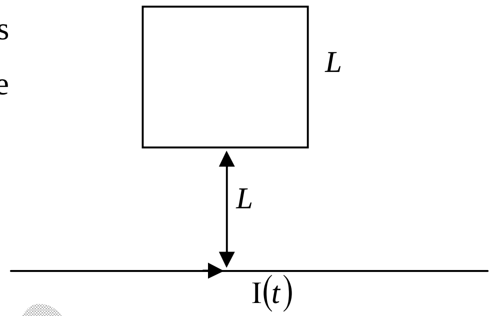
(a) 0

(b) $\frac{1}{2r}\omega \mu_0 n I_0 R^2 \cos \omega t$

(c) $\frac{1}{2}\omega \mu_0 n I_0 r \sin \omega t$

(d) $\frac{1}{2}\omega \mu_0 n I_0 r \cos \omega t$

Q5. An infinitely long wire carrying a current $I(t) = I_0 \cos\omega t$ is placed at a distance 'L' from a square loop of side 'L' as shown in figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is:



(a) $\frac{\mu_0}{4\pi}\cdot\frac{2LI_0\omega}{R}$ (b) $\frac{\mu_0}{4\pi}\cdot\frac{8LI_0\omega}{R}\ln 2$

(c) $\frac{\mu_0}{4\pi}\cdot\frac{4LI_0\omega}{R}\ln 2$ (d) $\frac{\mu_0}{4\pi}\cdot\frac{2LI_0\omega}{R}\ln 2$

Q6. A large circular coil of N turns and radius R carries a time varying current $I(t) = I_0 \sin\omega t$. A small circular coil of n turns and radius r $(r << R)$ is placed at the center of the large coil such that the coils are concentric and coplanar. The induced emf in the small coil

(a) Leads the current in the large coil by $\pi/2$

(b) Lags the current in the large coil by π

(c) is in phase with the current in the large coil

(d) Lags the current in the large coil by $\pi/2$

Q7. A thin conducting wire is bent into a circular loop of radius r and placed in a time dependent magnetic field of magnetic induction $\vec{B}(t) = B_0 e^{-\alpha t}\hat{z}$ *where* $B_0 > 0,\ \alpha > 0$ such that the plane of the loop is perpendicular to $\vec{B}(t)$. Then the induced emf in the loop is:

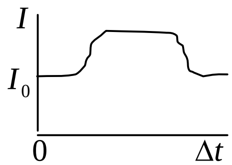
(a) $\pi r^2 \alpha B_0 e^{-\alpha t}$ (b) $\pi r^2 B_0 e^{-\alpha t}$

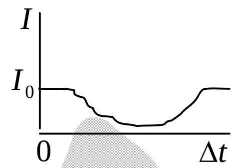
(c) $-\pi r^2 \alpha B_0 e^{-\alpha t}$ (d) $-\pi r^2 B_0 e^{-\alpha t}$

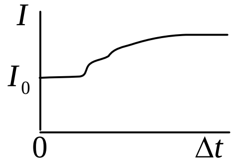
Q8. A square loop of wire, with sides of length L, lies in the first quadrant of the xy plane, with one corner at the origin. In this region there is a non-uniform time dependent magnetic field $\vec{B}(y,t) = Ky^3t^2\hat{z}$ (where K is a constant). Then the induced emf in the loop is

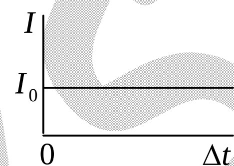
(a) $-\frac{1}{2}KtL^2$ (b) $-\frac{1}{4}KtL^4$ (c) $\frac{1}{2}KtL^5$ (d) $-\frac{1}{2}KtL^5$

Q9. A solenoid with an iron core is connected in series with a battery of emf V and it is found that a constant current I_0 passes through the solenoid. If at $t=0$, the iron core is pulled out from the solenoid quickly in a time Δt, which one of the following could be a correct description of the current passing through the solenoid?

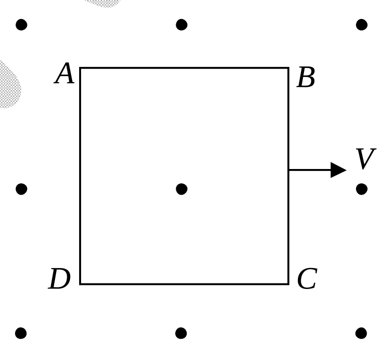
(a)


(b)


(c)


(d)


Q10. A metallic square loop $ABCD$ is moving in its own plane with velocity V in a uniform magnetic field perpendicular to its plane as shown in the figure. An electric field is induced.



(a) in AD, but not in BC

(b) in BC, but not in AD

(c) neither in AD nor in BC

(d) in both AD and BC

NAT (Numerical Answer Type)

Q11. A small loop of wire of area $A = 0.01\,m^2$, $N = 40$ turns and resistance $R = 40\,\Omega$ is initially kept in a uniform magnetic field B in such a way that the field is normal to the loop. When it is pulled out of the magnetic field a total charge of $Q = 2\times10^{-5}\,C$ flows through the coil. The magnetic field B is………… $\times10^{-3}\,Tesla$

Q12. A conducting circular loop is placed in a uniform magnetic field of $0.02\,Tesla$, with its plane perpendicular to the field. If the radius of the loop starts shrinking at a constant rate $1.0\,mm/sec$, then the magnitude of e.m.f. induced in the loop, at the instant when the radius is $4.0\,cm$ will be……… μV. (Answer must be an integer)

Q13. A long solenoid, of radius *a*, is driven by alternating current, so that the field inside is sinusoidal: $\vec{B}(t) = B_0\cos(\omega t)\hat{z}$. A circular loop of wire, of radius $a/2$ and resistance *R*, is placed inside the solenoid, and coaxial with it. Then the amplitude of current induced in the loop, as a function of time is $\alpha\dfrac{\pi a^2 B_0\omega}{R}$. Then the value of α is…………

MSQ (Multiple Select Questions)

Q14. Consider a solenoid of radius *R* with *n* turns per unit length, in which a time dependent current $I = I_0\sin\omega t$ (where $\omega R/c << 1$) flows. Then which of the following statements are true for magnitude of the electric field at a perpendicular distance r from the axis of symmetry?

(a) Electric field $|\vec{E}| = 0$ for $r > R$

(b) Electric field $|\vec{E}| = \dfrac{1}{2r}\omega\mu_0 n I_0 R^2\cos\omega t$ for $r > R$

(c) Electric field $|\vec{E}| = \dfrac{1}{2}\omega\mu_0 n I_0 r\sin\omega t$ for $r < R$

(d) Electric field $|\vec{E}| = \dfrac{1}{2}\omega\mu_0 n I_0 r\cos\omega t$ for $r < R$

Solutions

MCQ (Multiple Choice Questions)

Ans. 1: (b)

Ans. 2: (b)

Ans. 3: (a)

Ans. 4: (b)

Solution: $\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a};\qquad \left(\vec{B} = \mu_0 n I(t)\hat{z}\right)$.

$$\Rightarrow \left|\vec{E}\right| \times 2\pi r = -\mu_0 n \frac{dI}{dt}\int_{r'=0}^{R} 2\pi r' dr' = -\mu_0 n \times I_0 \omega \cos\omega t \times \frac{2\pi R^2}{2}$$

$$\Rightarrow \left|\vec{E}\right| = -\frac{1}{2r}\times \omega \mu_0 n I_0 R^2 \cos\omega t$$

Ans. 5: (d)

Solution: Magnetic flux $\phi = \int \vec{B}\cdot d\vec{a} = \frac{\mu_0 I(t)}{2\pi}\int_{L}^{2L}\frac{1}{r}\hat{\phi}\cdot L dr\hat{\phi} \Rightarrow \phi(t) = \frac{\mu_0 I(t) L}{2\pi}\ln 2$

$$\Rightarrow \varepsilon = -\frac{d\phi}{dt} = \frac{\mu_0}{2\pi}\cdot\frac{LI_0\omega\sin\omega t}{R}\ln 2 \qquad \because I(t) = I_0\cos\omega t$$

$$\Rightarrow \varepsilon = \frac{\mu_0}{4\pi}\cdot\frac{2LI_0\omega\sin\omega t}{R}\ln 2$$

Ans. 6: (a)

Solution: $\varepsilon = -\frac{d\phi}{dt} = -A\frac{dB}{dt} \propto -\frac{dI}{dt} \propto -\cos\omega t \propto -\sin\left(\frac{\pi}{2}-\omega t\right) \propto \sin\left(\omega t - \frac{\pi}{2}\right)$

Ans. 7: (a)

Solution: $\varepsilon = -\frac{d\phi}{dt} = \pi r^2 \times \alpha B_0 e^{-\alpha t}$

Ans. 8: (d)

Solution: $\phi = \int B dx dy = Kt^2\int_0^L dx\int_0^L y^3 dy = \frac{Kt^2L^5}{4} \Rightarrow \varepsilon = -\frac{d\phi}{dt} = -\frac{1}{2}KtL^5$

Ans. 9: (a)

Ans. 10: (d)

NAT (Numerical Answer Type)

Ans. 11: **2**

Solution: Magnetic flux through the loop $\phi = NBA$

Induced e.m.f $\varepsilon = -\frac{d\phi}{dt}$ and induced current $i = -\frac{1}{R}\frac{d\phi}{dt} = \frac{dQ}{dt} \Rightarrow -\frac{1}{R}d\phi = dQ$.

$$\Rightarrow \frac{1}{40}\times\left(40\times B\times 0.01\right) = 2\times10^{-5} \Rightarrow B = 2\times10^{-3}\ T$$

Ans. 12: **5.0**

Solution: $\varepsilon = -\frac{d\phi}{dt} = -\frac{BdA}{dt} = -\frac{\pi Bdr^2}{dr} = -\frac{2\pi Bdr}{dt}r$

$$\varepsilon = 2\times\frac{22}{7}\times2\times10^{-2}\times\left(1\times10^{-3}\right)\times4\times10^{-2} = 48\times10^{-7}V = 4.8\times10^{-6}V = 5.0\mu V$$

Ans. 13: **0.25**

Solution: Magnetic flux through the loop $\phi = \vec{B}.\vec{A} = B_0\cos\left(\omega t\right)\times\frac{\pi a^2}{4} = \frac{1}{4}\pi a^2 B_0\cos\left(\omega t\right)$

Induced emf $\varepsilon\left(t\right) = -\frac{d\phi}{dt} = \frac{1}{4}\pi a^2 B_0\omega\sin\omega t = \varepsilon_0\sin\omega t \Rightarrow \varepsilon_0 = \frac{1}{4}\pi a^2 B_0\omega$

Induced current $i\left(t\right) = \frac{\varepsilon\left(t\right)}{R} = \frac{\pi a^2 B_0\omega\sin\omega t}{4R} = i_0\sin\omega t \Rightarrow i_0 = \frac{\pi a^2 B_0\omega}{4R}$

MSQ (Multiple Select Questions)

Ans. 14: (b) and (d)

Solution: $\oint\vec{E}\cdot d\vec{l} = -\int\frac{\partial\overline{B}}{\partial t}\cdot d\vec{a};\qquad \left(\vec{B} = \mu_0 nI\left(t\right)\hat{z}\right).$

For $r > R \Rightarrow \left|\vec{E}\right|\times2\pi r = -\mu_0 n\frac{dI}{dt}\int_{r'=0}^{R}2\pi r'dr' = -\mu_0 n\times I_0\omega\cos\omega t\times\frac{2\pi R^2}{2}$

$$\Rightarrow\left|\vec{E}\right| = -\frac{1}{2r}\times\omega\mu_0 nI_0R^2\cos\omega t$$

For $r < R \Rightarrow \left|\vec{E}\right|\times2\pi r = -\mu_0 n\frac{dI}{dt}\int_{r'=0}^{r}2\pi r'dr' = -\mu_0 n\times I_0\omega\cos\omega t\times\frac{2\pi r^2}{2}$

$$\Rightarrow\left|\vec{E}\right| = -\frac{1}{2}\times\omega\mu_0 nI_0 r\cos\omega t$$