1. Basic Properties of Nuclei

1.1 Basic Nuclear Properties

An ordinary hydrogen atom has as its nucleus a single proton, whose charge is $+e$ and whose mass is 1836 times that of the electron. All other elements have nuclei that contain neutrons as well as protons. As its name suggests, the neutron is uncharged; its mass is slightly greater than that of the proton. Neutrons and protons are jointly called **nucleons**.

The **atomic number** of an element is the number of protons in each of its nuclei, which is the same as the number of electrons in a neutral atom of the element. Thus atomic number of hydrogen is 1, of helium 2, of lithium 3, and of uranium 92. All nuclei of a given element do not necessarily have equal numbers of neutrons. For instance, although over 99.9 percent of hydrogen nuclei are just single protons, a few also contain a neutron, and a very few two neutrons, along with the protons. The varieties of an element that differ in the numbers of neutrons their nuclei contain are called **isotopes**.

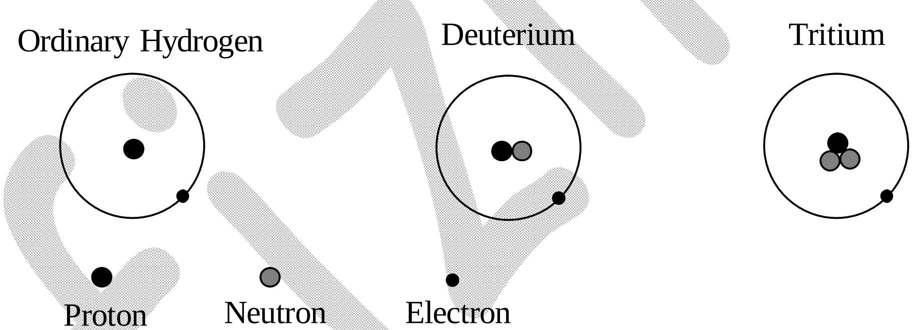

Figure: The isotope of hydrogen

The conventional symbols for nuclear species, or nuclides, follow the pattern ${}_Z^A X$, where

$X =$ Chemical symbol of the element

$Z =$ Atomic number of the element $=$ Number of protons in the nucleus

$A =$ Mass number of the nuclide $=$ Number of nucleons in the nucleus

Nuclear Terminology

- **Isotopes**

If two nuclei have same atomic number *Z* (proton), then they are called as isotopes.

Example: ${}^{13}_{6}C$ & ${}^{14}_{6}C$, ${}^{16}_{8}O$ & ${}^{17}_{8}O$ and ${}^{1}_{1}H$, ${}^{2}_{1}H$, ${}^{3}_{1}H$

- **Isotones**

If two nuclei have same neutron number *N* (proton), then they are called as isotones.

Example: ${}^{13}_{6}C$ and ${}^{14}_{7}N$

- **Isobars**

If two nuclei have same mass number *A*, then they are called as isobars.

Example: ${}^{14}_{6}C$ and ${}^{14}_{7}N$

- **Mirror nuclei**

Nuclei with same mass number *A* but with proton and neutron number interchanged and their difference is ± 1.

Example: ${}^{11}_{6}C$ & ${}^{11}_{5}B$ and ${}^{13}_{7}N$ & ${}^{13}_{6}C$

Atomic masses: Atomic masses refer to the masses of neutral atoms, not of bare nuclei. Thus an atomic mass always includes the masses of *Z* electrons. Atomic masses are expressed in **mass units** (*u*), which are so defined that the mass of a ${}^{12}_{6}C$ atom is exactly 12*u*. The value of mass unit is $1u = 1.66054\times10^{-27}\,kg \approx 931.4\,MeV$.

Some rest masses in various units are:

Particle	**Mass(kg)**	**Mass(*u*)**	**Mass(MeV/c^2)**
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	1.6750×10^{-27}	1.008665	939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}	0.511
${}_{1}H^{1}$	1.6736×10^{-27}	1.007825	938.79

1.1.1 Size and Density

Majority of atomic nuclei have spherical shape and only very few show departure from spherical symmetry. For spherically symmetrical nuclei, nuclear radius is given by

$$R = R_0 A^{1/3}$$

where A is the mass number and $R_0 = (1.2 \pm 0.1) \times 10^{-15} m \approx 1.2\ fm$.

R varies slightly from one nucleus to another but is roughly constant for $A > 20$.

The radius of ${}^{12}_{6}\text{C}$ nucleus is

$$R = (1.2)(12)^{1/3} \approx 2.7\ fm$$

Example: The radius of Ge nucleus is measured to be twice the radius of ${}^{9}_{4}\text{Be}$. How many nucleons are there in Ge nucleus?

Solution: $R = R_o (A)^{1/3} \Rightarrow R_{Ge} = 2R_{Be} \Rightarrow R_o (A)^{1/3} = 2R_o (9)^{1/3} \Rightarrow A = 72$

Nuclear Density

Assuming spherical symmetry, volume of nucleus is given by

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A.$$

Mass of one proton = 1.67×10^{-27} kg, Nuclear Mass = $A \times 1.67 \times 10^{-27}$ kg.

$$\text{Nuclear density} = \frac{A \times 1.67 \times 10^{-27}}{\frac{4}{3}\pi R_0^3 \times A} \approx 10^{17}\ \text{kg/m}^3$$

$$\text{Nuclear Particle Density} = \frac{\text{Nuclear Mass Density}}{\text{Nuclear Mass}} = \frac{10^{17}\ \text{Kg/m}^3}{1.67 \times 10^{-27}\ \text{Kg/Nucleon}}$$

$$= 10^{44}\ \text{Nucleons/m}^3$$

1.1.2 Spin and Magnetic Moment

Proton and neutrons, like electrons, are fermions with spin quantum numbers of $s=\frac{1}{2}$.

This means they have spin angular momenta $\vec{S}$ of magnitude

$$\left|\vec{S}\right|=\sqrt{s(s+1)}\hbar=\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar=\frac{\sqrt{3}}{2}\hbar$$

and spin magnetic quantum number of $m_s=\pm\frac{1}{2}$.

As in the case of electrons, magnetic moments are associated with the spins of protons and neutrons. In nuclear physics magnetic moments are expressed in **nuclear magnetons** (μ_N), where

Nuclear magneton $\mu_N=\frac{e\hbar}{2m_p}=5.051\times10^{-27}\,J/T=3.152\times10^{-8}\,eV/T$ where m_p is the proton mass.

In atomic physics we have defined **Bohr magneton** $\mu_B=\frac{e\hbar}{2m_e}$ where m_e is the electron mass.

The nuclear magneton is smaller than the Bohr magneton by the ratio of the proton mass to the electron mass which is 1836.

$$\frac{\mu_B}{\mu_N}=\frac{m_p}{m_e}=1836.$$

The spin magnetic moments of the proton and neutron have components in any direction of

Proton $\mu_{pz}=\pm2.793\mu_N$

Neutron $\mu_{nz}=\mp1.913\mu_N$

There are two possibilities for the signs of μ_{pz} and μ_{nz}, depending on whether m_s is $-\frac{1}{2}$ or $+\frac{1}{2}$. The $\pm$ sign is used for μ_{pz} because $\vec{\mu}_{pz}$ is in the same direction as the spin $\vec{S}$, whereas $\mp$ is used for μ_{nz} because $\vec{\mu}_{nz}$ is opposite to spin $\vec{S}$.

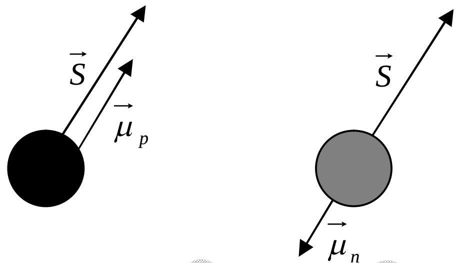

Figure: Spin magnetic moment ($\vec{\mu}$) and spin angular momentum ($\vec{S}$) directions for neutron and protons.

Note: For neutron, magnetic moment is expected to be zero as $e = 0$ but $\vec{\mu}_{nz} = \mp 1.913\mu_N$. At first glance it seems odd that the neutron, with no net charge, has spin magnetic moment. But if we assume that the neutron contains equal amounts of positive and negative charge, a spin magnetic moment arise if these charges are not uniformly distributed. Thus we can say that neutron has physical significance of negative charges because magnetic moment is opposite to that of its intrinsic spin angular momentum.

1.1.3 Angular Momentum of Nucleus

The hydrogen nucleus $^1_1\mathrm{H}$ consists of a single proton and its total angular momentum is given by $\left|\vec{S}\right| = \frac{\sqrt{3}}{2}\hbar$. A nucleon in a more complex nucleus may have orbital angular momentum due to motion inside the nucleus as well as spin angular momentum. The total angular momentum of such a nucleus is the vector sum of the spin and orbital angular momenta of its nucleons, as in the analogous case of the electrons of an atom.

When a nucleus whose magnetic moment has z component μ_z is in a constant magnetic field $\vec{B}$**,** the magnetic potential energy of the nucleus is

Magnetic energy $\qquad U_m = -\mu_z B$

The energy is negative when $\vec{\mu}_z$ is in the same direction as $\vec{B}$ and positive when $\vec{\mu}_z$ is opposite to $\vec{B}$. In a magnetic field, each angular momentum state of the nucleus is therefore split into components, just as in the Zeeman Effect in atomic electron states. Figure below shows the splitting when the angular momentum of the nucleus is due to the spin of a single proton.

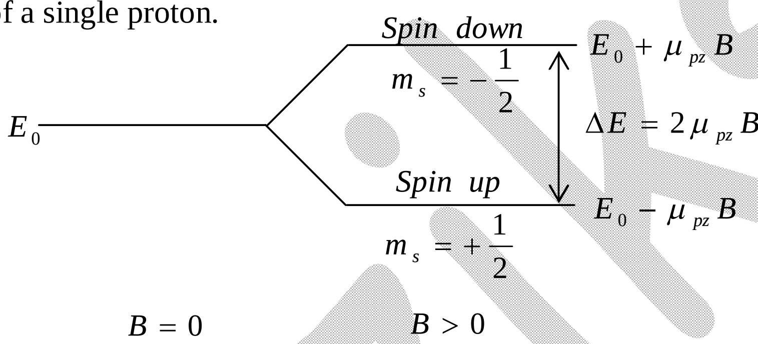

Figure: The energy levels of a proton in a magnetic field are split into spin-up and spin-down sublevels.

The energy difference between the sublevels is

$$\Delta E = 2\mu_{pz} B$$

A photon with this energy will be emitted when a proton in the upper state flips its spin to fall to the lower state. A proton in the lower state can be raised to upper one by absorbing a photon of this energy. The photon frequency ν_L that corresponds to ΔE is

Larmor frequency for photons $\qquad \nu_L = \frac{\Delta E}{h} = \frac{2\mu_{pz} B}{h}$

This is equal to the frequency with which a magnetic dipole precesses around a magnetic field.

1.1.4 Stable Nuclei

Not all combination of neutrons and protons form stable nuclei. In general, light nuclei $(A < 20)$ contain equal numbers of neutrons and protons, while in heavier nuclei the proportion of neutrons becomes progressively greater. This is evident in figure as shown below, which is plot of N versus Z for stable nuclides.

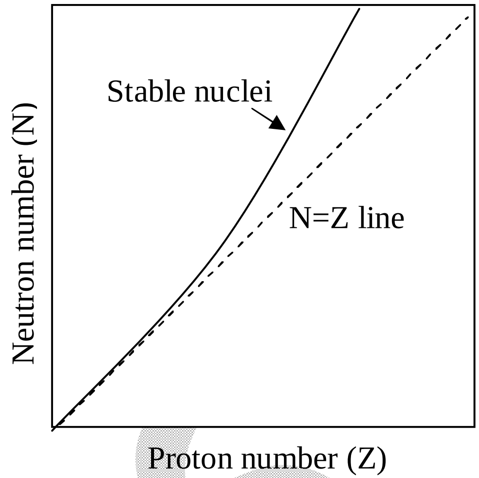

Figure: Neutron-proton diagram for stable nuclides.

The tendency for N to equal Z follows from the existence of nuclear energy levels. Nucleons, which have spin ½, obey exclusion principle. As a result, each energy level can contain two neutrons of opposite spins and two protons of opposite spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms are, to achieve configurations of minimum energy and therefore maximum stability. Thus the boron isotope $^{12}_{5}\text{B}$ has more energy than the carbon isotope $^{12}_{6}\text{C}$ because one of its neutrons is in a higher energy level, and $^{12}_{5}\text{B}$ is accordingly unstable. If created in a nuclear reaction, a $^{12}_{5}\text{B}$ nucleus changes by beta decay into a stable $^{12}_{6}\text{C}$ nucleus in a fraction of second.

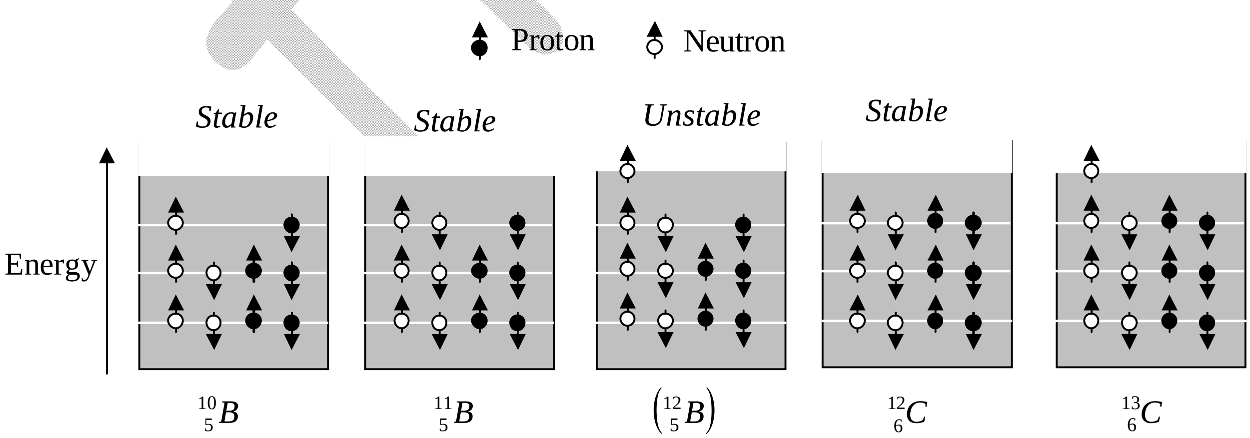

Figure: Simplified energy level diagrams of some boron and carbon isotopes.

The preceding argument is only part of the story. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so that an excess of neutrons, which produce only attractive forces is required for stability. Thus the curve departs more and more from $N = Z$ line as Z increases.

Sixty percent of stable nuclides have both even Z and even N; these are called **"even-even"** nuclides. Nearly all the others have either even Z and odd N (**"even-odd"** nuclides) or odd Z and even N (**"odd-even"** nuclides) with the numbers of both kinds being about equal. Only five stable **odd-odd** nuclides are known: ${}_{1}^{2}\mathrm{H}$, ${}_{3}^{6}\mathrm{Li}$, ${}_{5}^{10}\mathrm{B}$, ${}_{7}^{14}\mathrm{N}$ and ${}_{73}^{180}\mathrm{Ta}$. Nuclear abundances follow a similar pattern of favoring even numbers for Z and N.

These observations are consistent with the presence of nuclear energy levels that can each contain two particles of opposite spin. Nuclei with filled levels have less tendency to pick up other nucleons than those with partially filled levels and hence were less likely to participate in the nuclear reactions involved in the formation of elements.

Nuclear forces are limited in range, and as a result nucleons interact strongly only with their nearest neighbors. This effect is referred to as the **saturation** of nuclear forces. Because the coulomb repulsion of protons is appreciable throughout the entire nucleus, there is a limit to the ability of neutrons to prevent the disruption of large nucleus. This limit is represented by the bismuth isotope ${}_{83}^{209}\mathrm{Bi}$, which is the **heaviest stable** nuclide.

All nuclei with $Z > 83$ and $A > 209$ spontaneously transform themselves lighter ones through the emission of one or more alpha particles, which are ${}_{2}^{4}\mathrm{He}$ nuclei:

Alpha decay $${}_{Z}^{A}\mathrm{X} \rightarrow {}_{Z-2}^{A-4}\mathrm{Y} + {}_{2}^{4}\mathrm{He}$$

Since an alpha particle consists of two protons and two neutrons, an alpha decay reduces the Z and N of the original nucleus by two each. If the resulting daughter nucleus has either too small or too large a neutron/proton ratio for stability, it may beta-decay to a more appropriate configuration.

In negative beta decay, a neutron is transformed into a proton and an electron is emitted:

Negative beta decay $n^0 \to p^+ + e^-$

In positive beta decay, a proton becomes a neutron and a positron is emitted:

Positron emission $p^+ \to n^0 + e^+$

Thus negative beta decay decreases the proportion of neutrons and positive beta decay increases it. A process that competes with positron emission is the capture by a nucleus of an electron from its innermost shell. The electron is absorbed by a nuclear proton which is thereby transformed into neutron:

Electron Capture $p^+ + e^- \to n^0$

Figure below shows how alpha and beta decays enable stability to be achieved.

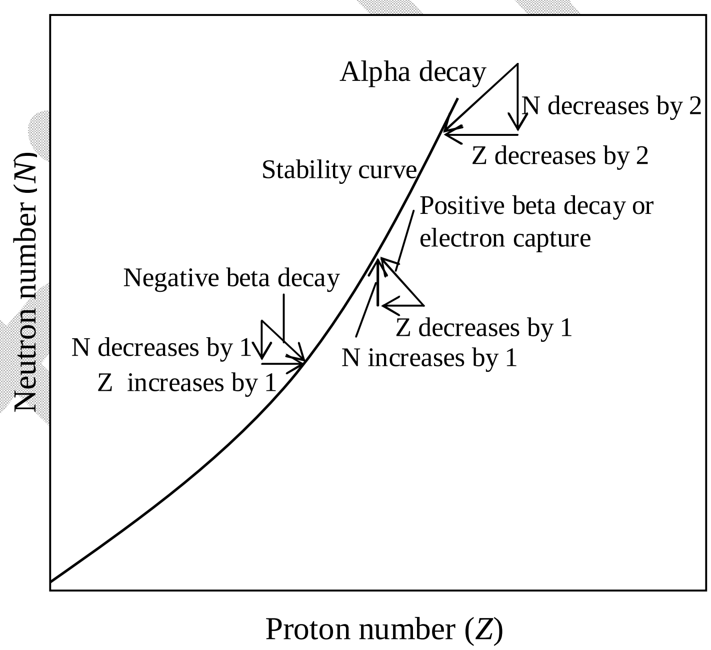

Figure: Alpha and beta decays permit an unstable nucleus to reach a stable configuration.

1.1.5 Binding Energy

When nuclear masses are measured, it is found that they are less than the sum of the masses of the neutrons and protons of which they are composed. This is in agreement with Einstein's theory of relativity, according to which the mass of a system bound by energy B is less than the mass of its constituents by B/c^2 (where c is the velocity of light).

The Binding energy B of a nucleus is defined as the difference between the energy of the constituent particles and of the whole nucleus. For a nucleus of atom ${}_Z^AX$,

$$B = \left[ZM_p + NM_N - {}_Z^AM\right]c^2 = [ZM_H + NM_N - M({}_Z^AX)]c^2$$

If mass is expressed in atomic mass unit

$$B = \left[ZM_p + NM_N - {}_Z^AM\right]\times 931.5\,MeV = [ZM_H + NM_N - M({}_Z^AX)]\times 931.5\,MeV$$

M_p : Mass of free proton, M_N : M$_N$: Mass of free neutron,

M_H : mass of hydrogen atom ${}_Z^AM$: mass of the nucleus,

Z : Number of proton, N : Number of neutron,

$M\left({}_Z^AX\right)$: mass of atom.

1.1.5.1 Binding Energy per Nucleon

The **binding energy per nucleon** for a given nucleus is found by dividing its total binding energy by the number of nucleon it contains. Thus binding energy per nucleon is

$$\frac{B}{A} = \frac{c^2}{A}\left[ZM_p + NM_N - {}_Z^AM\right] = \frac{c^2}{A}[ZM_H + NM_N - M({}_Z^AX)]$$

The binding energy per nucleon for ${}_1^2\text{H}$ is $\frac{2.224}{2} = 1.112\ \text{MeV}/\text{nucleon}$ and for ${}_{63}^{209}\text{Bi}$ it is $\frac{1640\ \text{MeV}}{209} = 7.8\ \text{MeV}/\text{nucleon}$.

Figure below shows the binding energy per nucleon against the number of nucleons in various atomic nuclei.

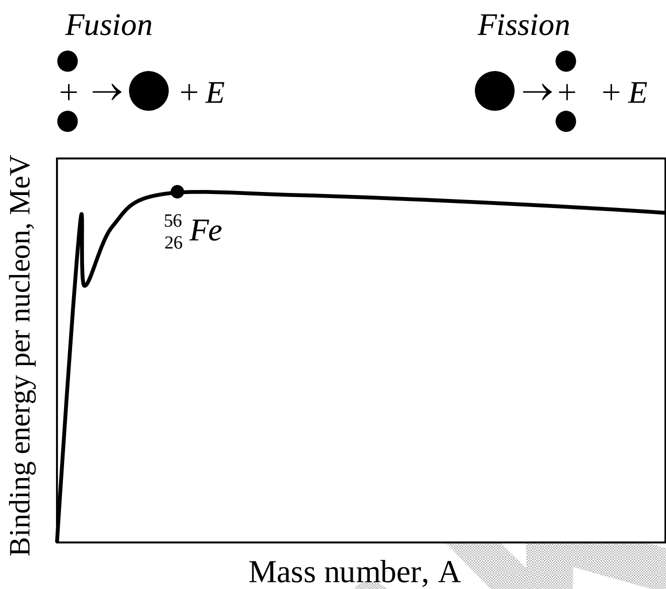

Figure: Binding energy per nucleon as function of mass number.

The greater the binding energy per nucleon, the more stable the nucleus is. The graph has the maximum of 8.8 MeV / nucleon when the number of nucleons is 56. The nucleus that has 56 protons and neutrons is $^{56}_{26}\text{Fe}$ an iron isotope. This is the most stable nucleus of them all, since the most energy is needed to pull a nucleon away from it.

Two remarkable conclusions can be drawn from the above graph.

(i) If we can somehow split a heavy nucleus into two medium sized ones, each of the new nuclei will have more binding energy per nucleon than the original nucleus did. The extra energy will be given off, and it can be a lot. For instance, if the uranium nucleus $^{235}_{92}\text{U}$ is broken into two smaller nuclei, the binding energy difference per nucleon is about 0.8 MeV. The total energy given off is therefore

$$\left(0.8\,\frac{\text{MeV}}{\text{nucleon}}\right)\left(235\ \text{nucleon}\right)=188\,\text{MeV}$$

This process is called as **nuclear fission**.

(ii) If we can somehow join two light nuclei together to give a single nucleus of medium size also means more binding energy per nucleon in the new nucleus. For instance, if two ${}_1^2\mathrm{H}$ deuterium nuclei combine to form a ${}_2^4\mathrm{He}$ helium nucleus, over 23 MeV is released. Such a process, called **nuclear fusion**, is also very effective way to obtain energy. In fact, nuclear fusion is the main energy source of the sun and other stars.

Example: The measured mass of deuteron atom $\left({}_1^2\mathrm{H}\right)$, Hydrogen atom $\left({}_1^1\mathrm{H}\right)$, proton and neutron is $2.01649\ u$, $1.00782\ u$, $1.00727\ u$ and $1.00866\ u$. Find the binding energy of the deuteron nucleus (unit $MeV/nucleon$).

Solution: Here $A=2,\ Z=1,\ N=1$

$$B.E.=[ZM_H+NM_N-M({}_1^2H)]\times 931.5\,MeV$$

$$=[1\times 1.00782+1\times 1.00866-2.01649]\times 931.5\,MeV$$

$$=[0.00238]\times 931.5\,MeV=2.224\,MeV$$

Example: The binding energy of the neon isotope ${}_{10}^{20}\mathrm{Ne}$ is 160.647 MeV. Find its atomic mass.

Solution: Here $A=10,\ Z=10,\ N=10$

$$\mathrm{M}\ ({}_Z^A\mathrm{X})=\left[\mathrm{ZM_H}+\mathrm{NM_N}\right]-\frac{\mathrm{B}}{931.5\,\mathrm{MeV/u}}$$

$$\mathrm{M}\ ({}_{10}^{20}\mathrm{Ne})=\left[10(1.00782)+10(1.00866)\right]-\frac{160.647}{931.5\,\mathrm{MeV/u}}=19.992\,\mathrm{u}$$

Example:

(a) Find the energy needed to remove a neutron from the nucleus of the calcium isotope ${}_{20}^{42}Ca$.

(b) Find the energy needed to remove a proton from this nucleus.

(c) Why are these energies different?

Given: atomic masses of ${}_{20}^{42}Ca=41.958622u$, ${}_{20}^{41}Ca=40.962278u$, ${}_{19}^{41}K=40.961825u$, and mass of ${}_0^1n=1.008665u$, ${}_1^1p=1.007276u$.

Solution:

(a) $^{42}_{20}Ca \rightarrow ^{41}_{20}Ca + ^{1}_{0}n$ **;**

Total mass of the $^{41}_{20}Ca\ and\ ^{1}_{0}n = 41.970943\ u$

Mass defect $\Delta m = 41.970943 - 41.958622 = 0.012321\, u$

So, B.E. of missing neutron $= \Delta m \times 931.5 = 11.48\ MeV$

(b) $^{42}_{20}Ca \rightarrow ^{41}_{19}K + ^{1}_{1}p$ **;**

Total mass of the $^{41}_{19}K\ and\ ^{1}_{1}p = 41.919101\ u$

Mass defect $\Delta m = 41.919101 - 41.958622 = 0.010479\, u$

So, B.E. of missing protron $= \Delta m \times 931.5 = 10.27\ MeV$

(c) The neutron was acted upon only by attractive nuclear forces whereas the proton was also acted upon by repulsive electric forces that decrease its binding energy.

1.1.6 Parity

Parity relates to the symmetry of the wave function that represents the system. If the wave function is unchanged, when the coordinates (x, y, z) are replaced by $(-x, -y, -z)$ then the system has a parity of $+1$. If the wave function has its sign changed, when the coordinates are reversed, then the system has parity of -1.

If we write

$$\psi(x, y, z) = P\psi(-x, -y, -z)$$

we can regard P as a quantum number characterizing ψ whose possible values are $+1$ and -1.

It has been observed that spatial part of ψ of a particle does not change its sign on reflection if the angular momentum quantum number "l" is even.

As a general rule **Parity** $= (-1)^l$

For a system of particles Parity is even if $\sum l$ even and Parity is odd if $\sum l$ odd.