4. Polarization of Light

Plane of Incidence

Incident ray, reflected ray, refracted ray and normal to incidence forms a plane that plane is plane of incidence.

Plane of Polarization

The plane passing through the direction of propagation and containing no vibration is called the "Plane of Polarization".

4.1 Production of Plane Polarized Light

Different methods of production of polarized light

(i) Polarization by reflection

(ii) Polarization by refraction

(iii) Polarization by selective absorption

(iv) Polarization by double refraction

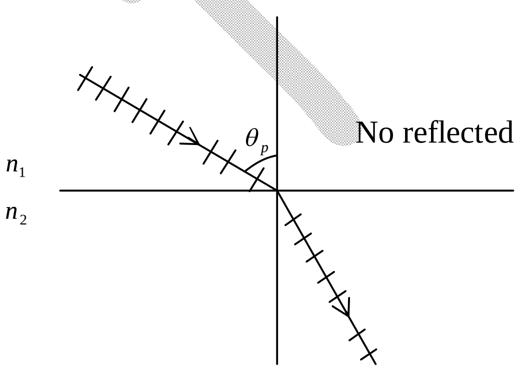
(v) Polarization by scattering

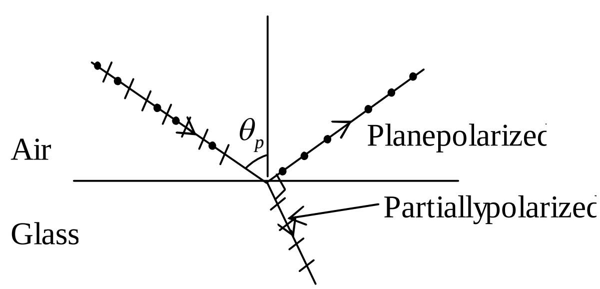
4.1.1 Polarization by Reflection

If a linearly polarized wave (Electric vector associated with the incident wave lies in the plane of incidence) is incident on the interface of two dielectrics with the angle of incidence equal to θ. If the angle of incidence θ is such that

$$\theta = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

then the reflection coefficient is zero.





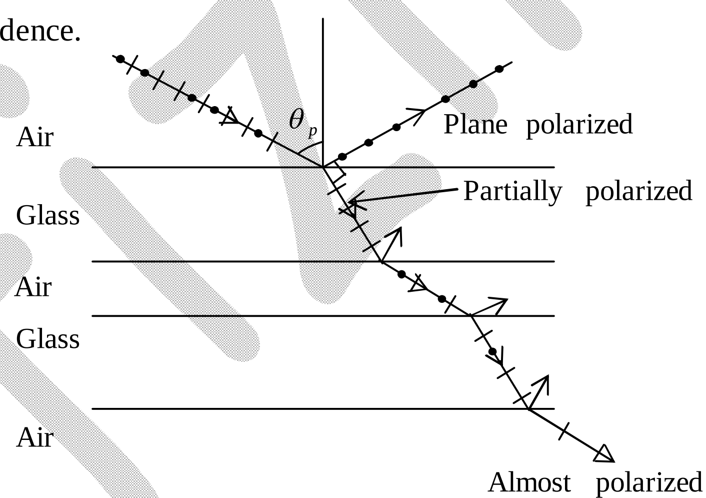
Thus if an unpolarized beam is incident with an angle of incidence equal to θ_p, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence.

Above equation is known as Brewster's law. The angle θ_p is known as the polarizing angle or the Brewster angle. At this angle, the reflected and the refracted rays are at right angle to each other i.e. $\theta_p + r = \frac{\pi}{2} \Rightarrow n = \frac{\sin\theta_p}{\sin r} = \frac{\sin\theta_p}{\cos\theta_p} \Rightarrow \tan\theta_p = n$

For the air-glass interface, $n_1 = 1$ and $n_2 \approx 1.5$ giving $\theta_p \approx 57^0$.

4.1.2 Polarization by Refraction

If an unpolarized beam is incident with an angle of incidence equal to θ_p, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence. The transmitted beam is partially polarized and if this beam is made to undergo several reflections, then the emergent beam is almost plane polarized with its electric vector in the plane of incidence.

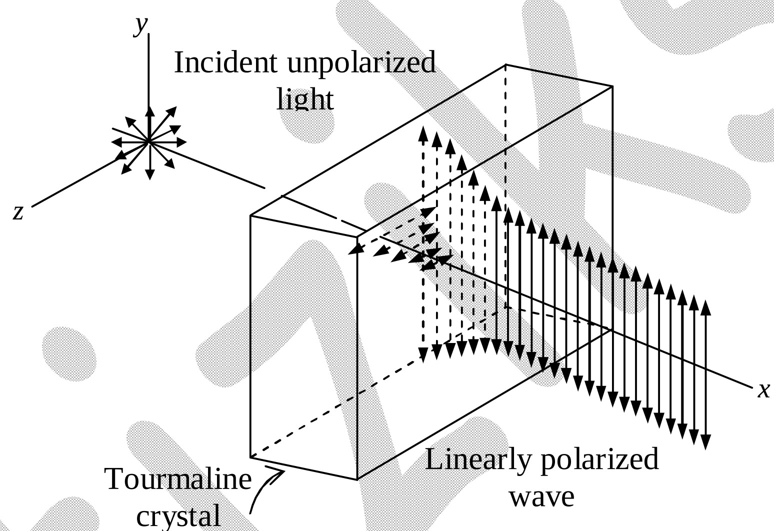


If I_p and I_s be the intensity of the parallel and perpendicular component in refracted light, then the degree of polarization is given by $P = \frac{I_p - I_s}{I_p + I_s} = \frac{m}{m + \left(\frac{2n}{1-n^2}\right)^2}$

where m is the number of plate and n is the refractive index.

4.1.3 Polarization by selective absorption

A simple method for eliminating one of the beams is through selective absorption; this property of selective absorption is known as dichroism. A crystal such as tourmaline has different coefficients of absorption for the two linearly polarized beams into which the incident beam splits up. Consequently, one of the beams gets absorbed quickly, and the other component passes through without much attenuation. Thus, if an unpolarized beam is passed through a tourmaline crystal, the emergent beam will be linearly polarized

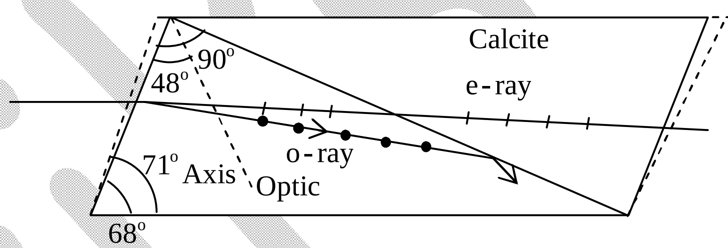


4.1.4 Polarization by Double Refraction

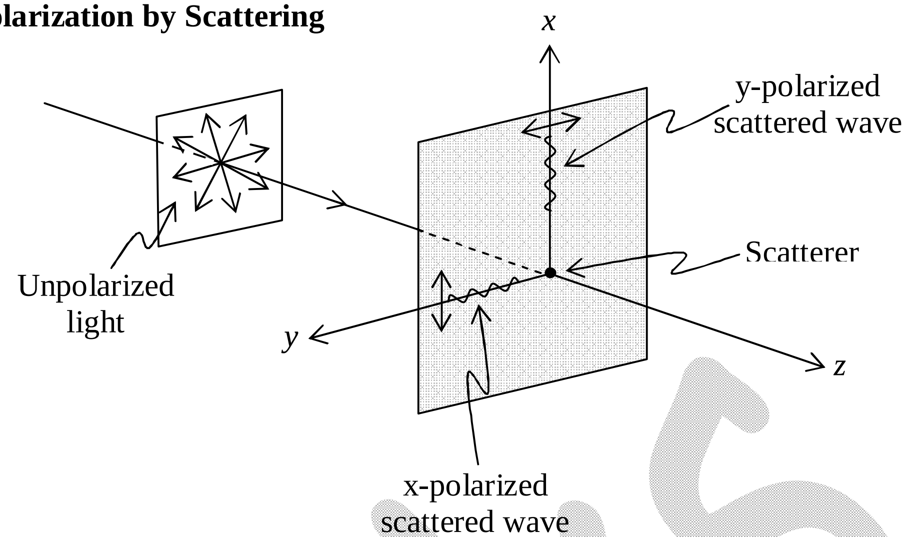
When a ray of unpolarized light passed through doubly refracting crystal (calcite or quartz), it split up into two refracted rays. One of the ray obeys the ordinary laws of refraction i.e. (n remains constant) and it is called 'ordinary ray' (o-ray). The other behaves in an extra ordinary way (i.e. n varies) and called 'extraordinary ray' (e-ray).

It is found that both the ordinary and extraordinary rays are plane-polarized having vibration perpendicular to each other.

If one can sandwich a layer of a material whose refractive index lies between the two, then for one of the beams, the incidence will be at a rarer medium and for the other it will be at a denser medium. This principle is used in a Nicol prism which consists of a calcite crystal cut in such a way that for the beam, for which the sandwiched material is a rarer medium, the angle of incidence is greater than the critical angle. Thus this particular beam will be eliminated by total internal reflection. Following figure shows a properly cut calcite crystal in which a layer of Canada balsam has been introduced so that the ordinary ray undergoes total internal reflection. The extraordinary component passes through, and the beam emerging from the crystal is linearly polarized.



4.1.5 Polarization by Scattering



When an unpolarized beam moving in z-axis, strike a gas atom than the electric field in unpolarized beam sets the electric charges in the gas atom in vibration. Since the E-field is in xy plane, therefore, the vibration will take place in xy plane only. These vibration can resolve in x and y direction and can said that two dipoles oscillating one in x and other in y-direction with the frequency of incident light. Since an oscillating dipole does not radiate in the direction of its own length.

Therefore, observer along x-axis will receive light component vibrating only in y-direction and along y-axis the component will vibrate only x axis. Hence observer will receive Plane Polarized light.

Blue colour of sky

Lord Rayleigh Formula: Intensity of light scattered from firie particles (dimension $\leq \lambda$) varies inversely as the fourth power of λ. i.e. $I \propto \frac{1}{\lambda^4}$

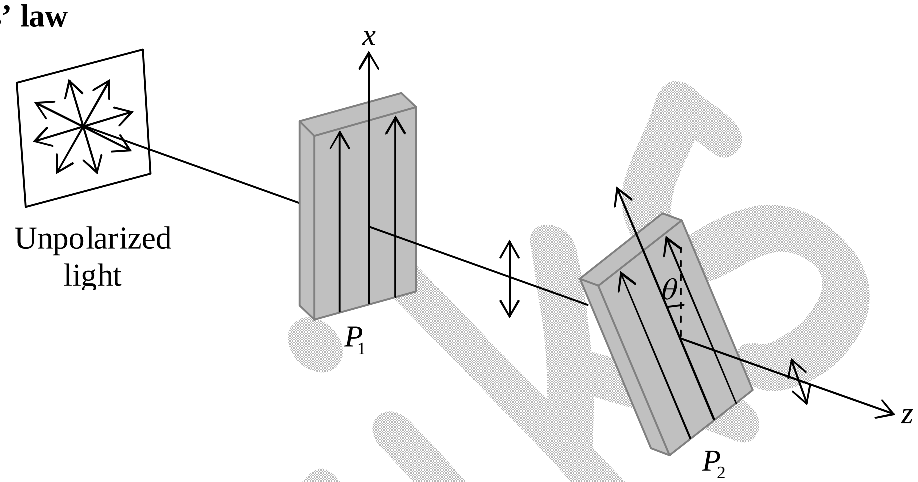
Scattered intensity of blue light is more than red $\because \lambda_{blue} < \lambda_{red}$.

Why blue scattered more: Natural frequency of bound electron in gas lies in ultra-violet region. Blue light closer to natural frequency than red light, therefore, blue is more effective in causing the electron to oscillate and thus it get more effectively scattered than red.

Red colour of sunrise and sunset

The path of the light through the atmosphere at sunrise and sunset is greatest. Since the violet & blue light is largely scattered and get removed and what we see is rest having red component.

4.2 Malus' law



An unpolarized light beam gets polarized after passing through the Polaroid P_1 which has a pass axis parallel to the x axis. When this x-polarized light beam incident on the second Polaroid P_2 whose pass axis makes an angle θ with the x axis, than the intensity of the emerging beam will vary as

$$I = I_0 \cos^2 \theta$$

where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is also along the x axis (i.e., when $\theta = 0$), above equation known as Malus' law.

Thus, if a linearly polarized beam is incident on a Polaroid and if the Polaroid is rotated about the z axis, then the intensity of the emergent wave will vary according to the above law.

4.3 Superposition of Two Disturbances and Production of Polarized Wave

4.3.1 Superposition of Two Waves with Parallel Electric Field

Let us consider the propagation of two linearly polarized electromagnetic waves (both propagating along the z axis) with their electric vectors oscillating along the x axis. The electric fields associated with the waves can be written in the form

$$E_1 = \hat{x}a_1 \cos\left(kz - \omega t + \theta_1\right) \quad (1)$$

$$E_2 = \hat{x}a_2 \cos\left(kz - \omega t + \theta_2\right) \quad (2)$$

where a_1 and a_2 represent the amplitudes of the waves, $\hat{x}$ represents the unit vector along the x axis, and θ_1 and θ_2 are phase constants. The resultant of these two waves is given by

$$E = E_1 + E_2 \quad (3)$$

which can always be written in the form

$$E = \hat{x}a \cos\left(kz - \omega t + \theta\right) \quad (4)$$

where $$a = \left[a_1^2 + a_2^2 + 2a_1a_2 \cos\left(\theta_1 - \theta_2\right)\right]^{\frac{1}{2}} \quad (5)$$

represents the amplitude of the wave. Equation (4) tells us that the resultant is also a linearly polarized wave with its electric vector oscillating along the same axis.

4.3.2 Superposition of Two Waves with Mutually Perpendicular Electric field

We next consider the superposition of two linearly polarized electromagnetic waves (both propagating along the z axis) but with their electric vectors oscillating along two mutually perpendicular directions. Thus, we may have

$$E_1 = \hat{x}a_1 \cos\left(kz - \omega t\right) \quad (6)$$

$$E_2 = \hat{y}a_2 \cos\left(kz - \omega t + \theta\right) \quad (7)$$

For $\theta = n\pi$, the resultant will also be a linearly polarized wave with its electric vector oscillating along a direction making a certain angle with the x axis; this angle will depend on the relative values of a_1 and a_2.

To find the state of polarization of the resultant field, we consider the time variation of the resultant electric field at an arbitrary plane perpendicular to the z axis which we may, without any loss of generality, assume to be $z=0$.

If E_x and E_y represent the x and y components of the resultant field $E=(E_1+E_2)$, then

$$E_x = a_1 \cos \omega t \tag{8}$$

and
$$E_y = a_2 \cos(\omega t - \theta) \tag{9}$$

where we have used Equations (6) and (7) with $z=0$.

For $\theta = n\pi$ the above equations simplify to

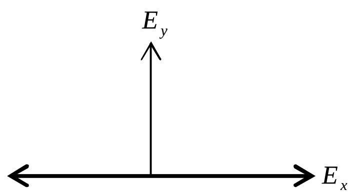
$$E_x = a_1 \cos \omega t \quad \text{and} \quad E_y = (-1)^n a_2 \cos \omega t \tag{10}$$

from which we obtain
$$\frac{Ey}{E_x} = \pm \frac{a_2}{a_1} \text{ (independent of } t\text{)} \tag{11}$$

where the upper and lower signs correspond to n even and n odd, respectively. In the $E_x E_y$ plane, Eq. (11) represents a straight line; the angle ϕ that this line makes with the E_x axis depends on the ratio $\frac{a_2}{a_1}$. In fact
$$\phi = \tan^{-1}\left(\pm \frac{a_2}{a_1}\right) \tag{12}$$

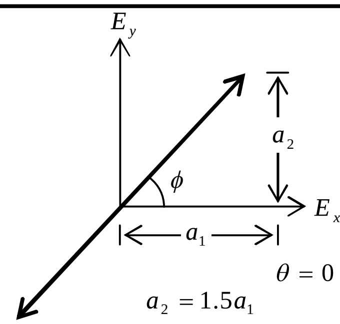
The condition $\theta = n\pi$ implies that the two vibrations are either in phase $(n = 0, 2, 4....)$ or out of phase $(n = 1, 3, 5, ...)$. Thus, the superposition of two linearly polarized electromagnetic waves with their electric fields at right angles to each other and oscillating in phase is again a linearly polarized wave with its electric vector, in general, oscillating in a direction which is different from the fields of either of the two waves.

Following figures shows the plot of the resultant field corresponding to Eq. (10) for various values of $\frac{a_2}{a_1}$. The tip of the electric vector oscillates (with angular frequency ω) along the thick lines shown in the figure. The equation of the straight line is given by Eq. (11).



$a_2 = 0$

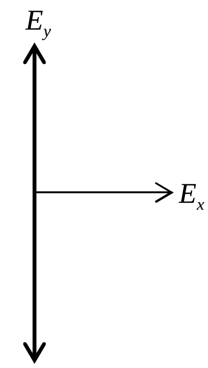
(i)



$\theta = 0$

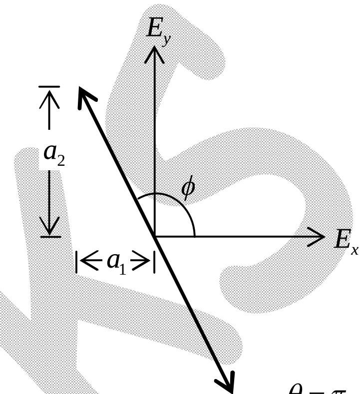
$a_2 = 1.5a_1$

(ii)



$a_1 = 0$

(iii)



$\theta = \pi$

$a_2 = 1.5a_1$

(iv)

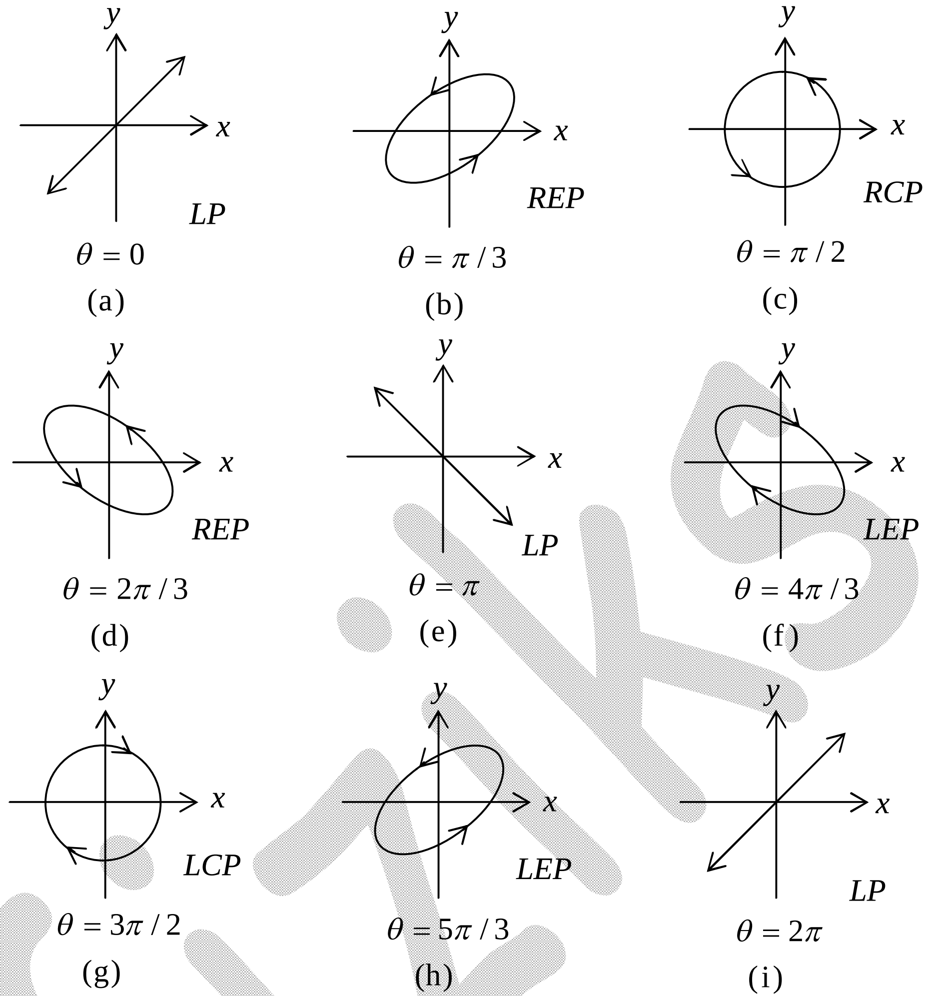
For $\theta \neq n\pi\,(n = 0,1,2,...)$, the resultant electric vector does not, in general, oscillate along a straight line.

We first consider the simple case corresponding to $\theta = \frac{\pi}{2}$ with $a_1 = a_2$. Thus,

$E_x = a_1 \cos \omega t$ (13) and $E_y = a_1 \cos \omega t$ (14)

If we plot the time variation of the resultant electric vectors whose x and y components are given by Eqs. (13) and (14), we find that the tip of the electric vector rotates on the circumference of a circle (of radius a_1) in the counterclockwise direction as shown in Fig. (c) below, and the propagation is in the $+z$ direction which is coming out of the page. Such a wave is known as a right circularly polarized wave (usually abbreviated as a RCP wave). That the tip of the resultant electric vector should lie on the circumference of circle is also obvious from the fact that

$$E_x^2 + E_y^2 = a_1^2 \qquad \text{(Independent of } t\text{)}$$



For $\theta = \frac{3\pi}{2}$, $\quad E_x = a_1 \cos \omega t \quad$ (15) $\quad$ and $\quad E_y = -a_1 \sin \omega t \quad$ (16)

which would also represent a circularly polarized wave; however, the electric vector will rotate in the clockwise direction [Fig. (g)]. Such a wave is known as a left circularly polarized wave (usually abbreviated as a LCP wave). For $\theta \neq \frac{m\pi}{2}\left(m = 0, 1, 2, ...\right)$, the tip of the electric vector rotates on the circumference of an ellipse. As can be seen from the figure, this ellipse will degenerate into a straight line or a circle when θ becomes an even or an odd multiple of $\frac{\pi}{2}$. In general, when $a_1 \neq a_2$, one obtains an elliptically polarized wave which degenerates into a straight line for $\theta = 0, \pi, 2\pi, ...$ etc.

4.4 The Phenomenon of Double Refraction

When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams as shown in Fig. (a). The beam which travels undeviated is known as the ordinary ray (usually abbreviated as the $o-$ray) and obeys Snell's laws of refraction. On the other hand, the second beam, which in general does not obey Snell's laws, is known as the extraordinary ray (usually abbreviated as the $e-$ray).

The appearance of two beams is due to the phenomenon of double refraction, and a crystal such as calcite is usually referred to as a double refracting crystal. If we put a Polaroid PP' behind the calcite crystal and rotate the Polaroid (about NN'), then for two positions of the Polaroid (when the pass axis is perpendicular to the plane of the paper) the e - ray will be completely blocked and only the o-ray will pass through.

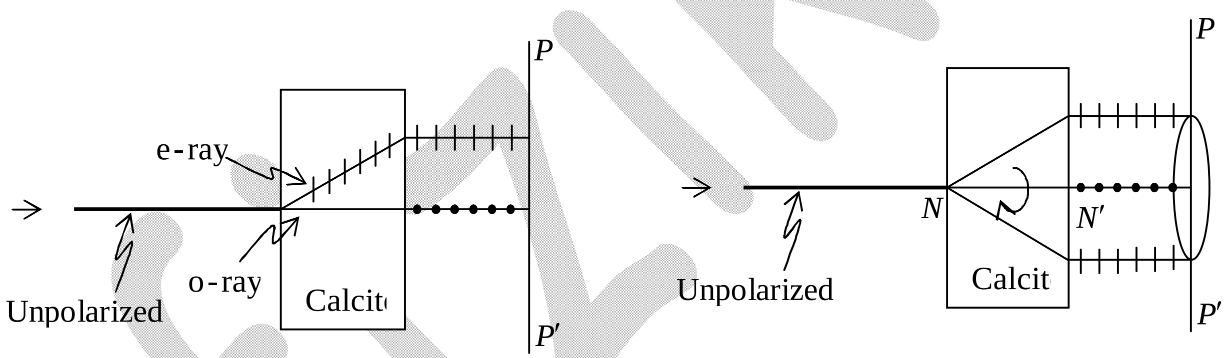


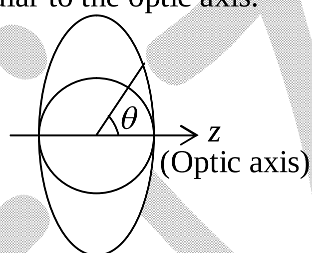
Fig (a) When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams. (b) If we rotate the crystal about NN' then the e-ray will rotate about NN'.

On the other hand, when the pass axis of the Polaroid is in the plane of the paper (i.e., along the line PP'), then the o - ray will be completely blocked and only the e-ray will pass through. Further, if we rotate the crystal about NN' then the e -ray will rotate about the axis [see Fig. (b)].

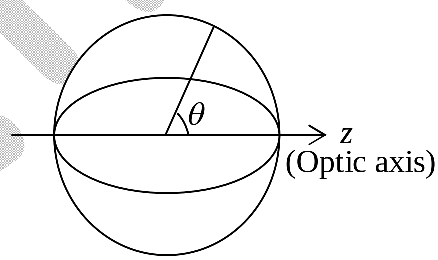
The velocity of the ordinary ray is the same in all directions, the velocity of the extraordinary ray is different in different directions; a substance (such as as calcite, quartz) which exhibits different properties in different directions is called an anisotropic substance. Along a particular direction (fixed in the crystal), the two velocities are equal; this direction is known as the optic axis of the crystal. In a crystal such as calcite, the two rays have the same speed only along one direction (which is the optic axis); such crystals are known as uniaxial crystals. The velocities of the ordinary and the extraordinary rays are given by the following equations:

$$v_{ro} = \frac{c}{n_o} \text{(Ordinary ray)} \quad \text{and} \quad \frac{1}{v_{r_e}^2} = \frac{\sin^2\theta}{\left(\frac{c}{n_e}\right)^2} + \frac{\cos^2\theta}{\left(\frac{c}{n_o}\right)^2} \text{ (extraordinary)}$$

where n_o and n_e are refractive index for O - ray and e - ray and θ is the angle that the ray makes with the optic axis; we have assumed the optic axis to be parallel to the z axis. Thus, $\frac{c}{n_o}$ and $\frac{c}{n_e}$ are the velocities of the extraordinary ray when it propagates parallel and perpendicular to the optic axis.



Negative crystal (Calcite)
(a)

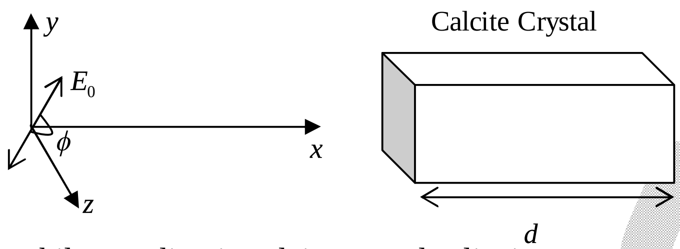


Positive crystal (Quartz)
(b)

Fig. (a) In a negative crystal, the ellipsoid of revolution (which corresponds to the extra ordinary ray) lies outside the sphere; the sphere corresponds to the ordinary ray. (b) In a positive crystal, the ellipsoid of revolution (which corresponds to the extraordinary ray) lies inside the sphere.

4.5 Quarter Wave Plate and Half Wave Plate

Let electric field vector (of amplitude E_0) associated with the incident linearly polarized beam makes an angle ϕ with optic axis which is parallel to z-axis and incident on calcite crystal of thickness d whose optic axis is parallel to the surface



Such beam while traveling in calcite crystal splits into two components. The z-axis whose amplitude is $E_0 \cos\phi$ passes through as an extraordinary ray (e-ray) propagates with velocity c/n_e. The y-axis whose amplitude is $E_0 \sin\phi$ passes through as an ordinary ray (o-ray) propagates with velocity c/n_o.

Since $n_o \neq n_e$ the two beams will propagate with different velocities, thus when they come out of the crystal, they will not be in phase. Let on the plane $x = 0$, the beam is incident then

$$E_y = E_0 \sin\phi \cos(kx - \omega t) \quad \text{and } E_z = E_0 \cos\phi \cos(kx - \omega t).$$

Thus at $x = 0$, we have $\quad E_y = E_0 \sin\phi \cos\omega t \quad$ and $E_z = E_0 \cos\phi \cos\omega t$.

Inside crystal the two components will be

$$E_y = E_0 \sin\phi \cos(n_o kx - \omega t) \quad \text{and } E_z = E_0 \cos\phi \cos(n_e kx - \omega t).$$

If thickness of the crystal is d then on emerging surface we have

$$E_y = E_0 \sin\phi \cos(n_o kd - \omega t) \quad \text{and } E_z = E_0 \cos\phi \cos(n_e kd - \omega t).$$

Thus the phase difference between e-ray and o-ray is $\delta = kd(n_o - n_e)$.

If phase difference is $\delta = \pi/2$, then $d = \dfrac{\lambda}{4(n_o - n_e)}$ (Quarter Wave Plate)

If phase difference is $\delta = \pi$, then $d = \dfrac{\lambda}{2(n_o - n_e)}$ (Half Wave Plate)

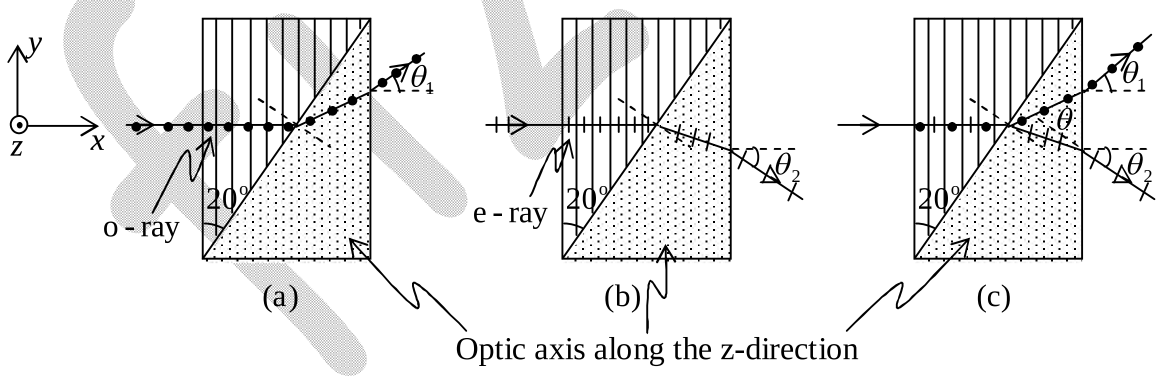
4.6 Wollaston Prism

A Wollaston prism is used to produce two linearly polarized beams. It consists of two similar prisms (calcite) with the optic axis of the first prism parallel to the surface and the optic axis of the second prism parallel to the edge of the prism as shown below. Let us first consider the incidence of a z polarized beam as shown in Fig. (a). The beam will propagate as an $o-$ ray in the first prism (because the vibrations are perpendicular to the optic axis) and will see the refractive index n_0. When this beam enters the second prism, it will become an e-ray and will see the refractive index n_e. For calcite $n_o > n_e$ and the ray will bend away from the normal. Since the optic axis is normal to the plane of paper, the refracted ray will obey Snell's laws, and the angle of refraction will be given by

$$n_0 \sin 20^o = n_e \sin r_1$$

where we have assumed the angle of the prism to be 20^o. Assuming $n_o \approx 1.658$ and $n_e \approx 1.486$, we readily get $r_1 \approx 22.43^o$

Thus the angle of incidence at the second surface is $i_1 = 22.43^o - 20^o = 2.43^o$. The output angle θ_1 is given by $n_e \sin 2.43^o = \sin\theta_1 \Rightarrow \theta_1 = 3.61^o$.



We next consider the incidence of a y - polarized beam as shown in Fig. (b). The beam will propagate as an e-ray in the first prism and as an o-ray in the second prism. The angle of refraction is now given by

$$n_e \sin 20 = n_o \sin r_2 \Rightarrow r_2 \approx 17.85^o$$

Thus the angle of incidence at the second interface is

$$i_2 = 20^o - 17.85^o = 2.15^o$$

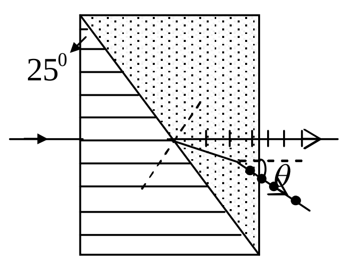
The output angle θ_2 is given by

$$n_o \sin 2.15^o = \sin\theta_2 \Rightarrow \theta_2 \approx 3.57^o$$

Thus, if an unpolarized beam is incident on the Wollaston prism, the angular separation between the two orthogonally polarized beams is $\theta = \theta_1 + \theta_2 \approx 7.18^o$.

4.7 Rochon Prism

We next consider the Rochon prism which consists of two similar prisms of (say) calcite; the optic axis of the first prism is normal to the face of the prism while the optic axis of the second prism is parallel to the edge as shown in figure. Now, in the first prism both beams will see the same refractive index n_o; this follows from the fact that the ordinary and extraordinary waves travel with the same velocity $\left(\frac{c}{n_o}\right)$ along the optic axis of the crystal. When the beam enters the second crystal, the ordinary ray (whose D is normal to the optic axis) will see the same refractive index and go undeviated as shown in figure. On the other hand, the extraordinary ray (whose D is along the optic axis) will see the refractive index n_e and will bend away from the normal. We assume the angle of the prism to be 25°. The angle of refraction will be determined from

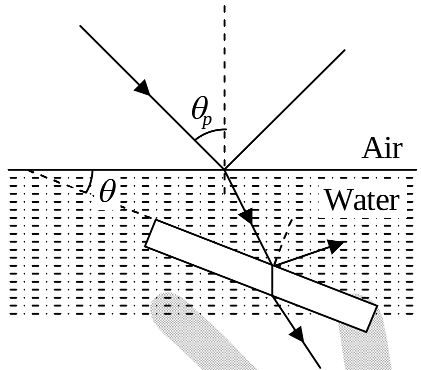


$$n_o \sin 25^\circ = n_e \sin r$$

Thus $\sin r = \frac{n_o}{n_e}\sin 25^\circ = \frac{1.658}{1.486}\times 0.423 \approx 0.472 \Rightarrow r = 28.2^\circ$

Therefore the angle of incidence at the second surface will be $28.2^\circ - 25^\circ = 3.2^\circ$. The emerging angle will be given by $\sin\theta = n_e \sin\left(3.2^\circ\right) \Rightarrow \theta \approx 4.8^\circ$

Example: Light strikes a water surface at the polarizing angle. The part of the beam refracted into the water strikes a submerged glass slab (index of refraction, 1.50), as shown in Figure. The light reflected from the upper surface of the slab is completely polarized. Find the angle between the water surface and the glass slab

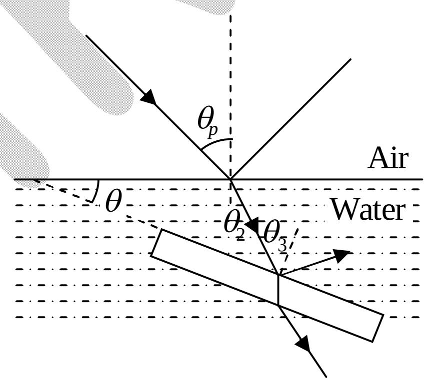


Solution: For the air-to-water interface,

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \left(\theta_p = 53.1^{\circ}\right)$$

and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1^{\circ}}{1.33}\right) = 36.9^{\circ}$$



For the water to glass interface,

$$\tan\theta_p = \tan\theta_3 = \frac{n_{\text{glass}}}{\text{n}_{\text{water}}} = \frac{1.50}{1.33}$$

$$\Rightarrow \theta_3 = 48.4^{\circ}$$

The angle between surfaces is $\theta = \theta_3 - \theta_2 = 11.5^{\circ}$

Example: Plane- polarized light is incident on a single polarizing disk with the direction of E_0 parallel to the direction of the transmission axis. Through what angle should the disk be rotated so that the intensity in the transmitted beam is reduced by a factor of (a) 3.00, (b) 5.00, (c) 10,0?

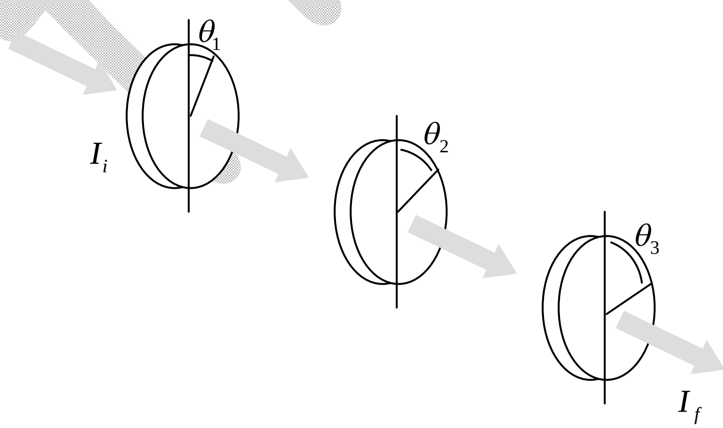
Solution: $I = I_{max}\cos^2\theta \Rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{I}{I_{max}}}$

(a) $\frac{I}{I_{max}} = \frac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{1}{3.00}} = 54.7^o$

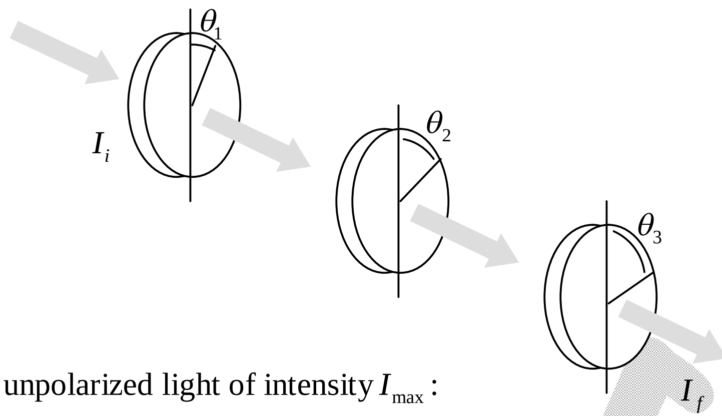
(b) $\frac{I}{I_{max}} = \frac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{1}{5.00}} = 63.4^o$

(c) $\frac{I}{I_{max}} = \frac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{1}{10.0}} = 71.6^o$

Example: In figure below, suppose that the transmission axes of the left and right polarizing disks are perpendicular to each other, also, let center disk be rotated on the common axis with an angular speed ω. Show that if unpolarized light is incident on the left disk with intensity I_{max} of the beam emerging from right disk is $I = \frac{1}{16} I_{max}\left(1-\cos 4\omega t\right)$



Solution:



For incident unpolarized light of intensity $I_{\max}$:

After transmitting 1st disk: $I = \frac{1}{2} I_{\max}$

After transmitting 2nd disk: $I = \frac{1}{2} I_{\max} \cos^2\theta$

After transmitting 3rd disk: $I = \frac{1}{2} I_{\max} \cos^2\theta \cos^2\left(90^0 - \theta\right)$

Where the angle between the first and second disk is $\theta = \omega t$

Using trigonometric identities $\cos^2\theta = \frac{1}{2}\left(1 + \cos 2\theta\right)$

And $\cos^2\left(90^0 - \theta\right) = \sin^2\theta = \frac{1}{2}\left(1 - \cos 2\theta\right)$

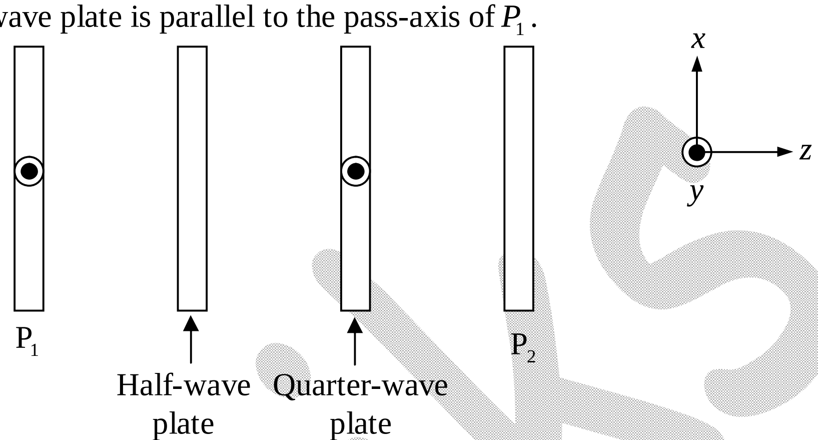
We have $I = \frac{1}{2} I_{\max}\left[\frac{\left(1+\cos 2\theta\right)}{2} . \frac{\left(1-\cos 2\theta\right)}{2}\right]$

$I = \frac{1}{8} I_{\max}\left(1 - \cos^2 2\theta\right) = \frac{1}{8} I_{\max}\left(\frac{1}{2}\right)\left(1 - \cos 4\theta\right)$

since $\theta = \omega t$, the intensity of the emerging beam is given by

$$I = \frac{1}{16} I_{\max}\left[1 - \cos\left(4\omega t\right)\right]$$

Example: A half-wave plate and a quarter-wave plate are placed between a polarizer P_1 and an analyzer P_2. All of these are parallel to each other and perpendicular to the direction of propagation of unpolarized incident light (see the figure). The optic-axis of the half-wave plate makes an angle of 30^0 with respect to the pass-axis of P_1 and that of the quarter-wave plate is parallel to the pass-axis of P_1.



(a) Determine the state of polarization for the light after passing through (i) the half-wave plate and (ii) the quarter-wave plate.

(b) What should be the orientation of the pass-axis of P_2 with respect to that of P_1 such that the intensity of the light emerging from P_2 is maximum?

Solution :(a) After passing through HWP the incident ray will split into an o-ray of amplitude $E_0 \sin 30^0 = E_0/2$ and of e-ray of amplitude $E_0 \cos 30^0 = E_0\sqrt{3}/2$. After emerging out of HWP they will have phase difference of π. Superposition of these o- and e-ray will produce linearly polarized light at the output of HWP.

The electric field components of o-ray and e-ray at the output of HWP is

$$\vec{E}_{o-ray} = \left(\frac{E_0}{2}\cos 60^0 \hat{i} - \frac{E_0}{2}\cos 30^0 \hat{k}\right)\sin(\omega t - kz) \quad = \left(\frac{E_0}{4}\hat{i} - \frac{\sqrt{3}E_0}{4}\hat{j}\right)\sin(\omega t - kz)$$

$$\vec{E}_{e-ray} = \left(\frac{\sqrt{3}E_0}{2}\cos 30^0 \hat{i} + \frac{\sqrt{3}E_0}{2}\cos 60^0 \hat{k}\right)\sin(\omega t - kz + \pi)$$

$$= \left(\frac{3E_0}{4}\hat{i} + \frac{\sqrt{3}E_0}{4}\hat{k}\right)\sin(\omega t - kz + \pi)$$

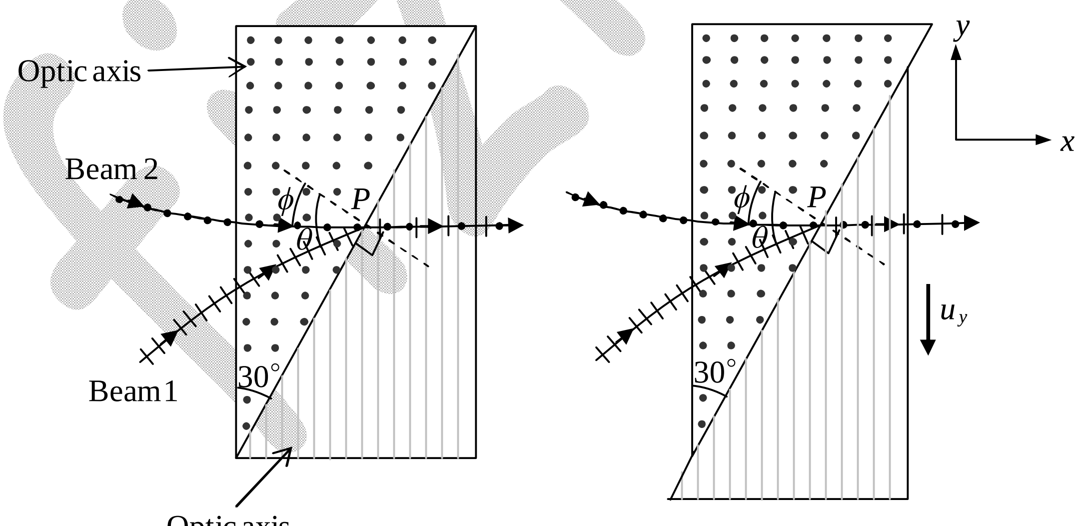
The electric field components of linearly polarized light at the output of HWP is

$$\vec{E} = E_0 \sin(\omega t - kz)\hat{i}$$

The optic axis of QWP is parallel to the P_1 i.e along x-axis. The electric field component of incident linearly polarized light makes zero angle with the optic axis as a result the incident light will simply pass through QWP as a e-ray. Therefore at the output of the QWP light will be linearly polarized with the equation $\vec{E} = E_0 \sin(\omega t - kz)\hat{i}$

(b) The orientation of the pass axis of P_2 should be parallel to the pass axis of P_1 to allow maximum intensity of light to pass through P_2.

Example: Two orthogonally polarized beams (each of wavelength 0.5 μm and with polarization marked in the figure) are incident on a two-prism assembly and emerge along x-direction, as shown. The prisms are of identical material and n_o and n_e are the refractive indices of the o-ray and e-ray, respectively. Use $\sin\phi = \dfrac{\sin\theta}{3}$, and $n_o = \dfrac{\sqrt{3}+1}{4}$.



(a) Find the value of θ and n_e. (b) If the right hand side prism starts sliding down with the vertical component of the velocity $u_y = 1\mu m/s$, what would be the minimum time after which the state of polarization of the emergent beam would repeat itself?

Solution:

(a) The beam will propagate as an $o-$ray in the first prism (because the vibrations are perpendicular to the optic axis) and will see the refractive index n_0. When this beam enters the second prism, it will become an $e-$ray and will see the refractive index n_e. For calcite $n_o > n_e$ and the ray will bend away from the normal. Since the optic axis is normal to the plane of, the refracted ray will obey Snell's laws, and the angle of refraction will be given by $n_0 \sin 30^0 = n_e \sin\theta$

We next consider the incidence of a second beam. The beam will propagate as an $e-$ray in the first prism and as an $o-$ray in the second prism. The angle of refraction is now given by $n_e \sin 30^0 = n_0 \sin\phi$ $\Rightarrow n_e \frac{1}{2} = \frac{\sqrt{3}+1}{4} \times \frac{\sin\theta}{3} \Rightarrow n_e = \frac{\sqrt{3}+1}{2} \times \frac{\sin\theta}{3}$

From 1st equation $n_0 \sin 30^0 = n_e \sin\theta \Rightarrow \sin\theta = \frac{n_0}{2n_e} = \frac{\sqrt{3}+1}{4} \times \frac{3\times 2}{\left(\sqrt{3}+1\right)\sin\theta} \times \frac{1}{2}$

$$\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^0$$

Hence $n_e = \frac{\sqrt{3}+1}{2} \times \frac{\sin\theta}{3} = \frac{\sqrt{3}+1}{2} \times \frac{\sin 60^0}{3} = \frac{\sqrt{3}+1}{2} \times \frac{\sqrt{3}/2}{3} = \frac{1}{4}\left(1+\frac{1}{\sqrt{3}}\right)$

(b) The state of polarization will repeat if change in phase difference (δ) is π.

The relation between phase difference and path difference (Δ) is $\delta = \frac{2\pi}{\lambda}\Delta$

Where, $\Delta = (n_0 - n_e)d = (n_0 - n_e)\times 10^{-6} \times t$

Thus $\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}(n_0 - n_e)\times 10^{-6} \times t = \pi \Rightarrow t = \frac{1}{4(n_0 - n_e)}$

While $n_0 - n_e = \frac{\sqrt{3}+1}{4} - \frac{\sqrt{3}+1}{4\sqrt{3}} = \frac{1}{2\sqrt{3}}$. Thus $t = \frac{1}{4(n_0 - n_e)} = \frac{1}{4} \times 2\sqrt{3} = \frac{\sqrt{3}}{2} = 0.85\,\text{sec}$

4.8 Analysis of Polarized Light

(i) Linearly polarized (ii) Circularly polarized

(iii) Elliptically polarized (iv) Unpolarized

(v) Mixture of linearly polarized and unpolarized

(vi) Mixture of circularly polarized and unpolarized

(vii) Mixture of elliptically polarized and unpolarized light

If we introduce a Polaroid in the path of the beam and rotate it about the direction of propagation, then one of the following three possibilities can occur:

1. If there is complete extinction at two positions of the polarizer, then the beam is linearly polarized.

2. If there is no variation of intensity, then the beam is unpolarized or circularly polarized or a mixture of unpolarized and circularly polarized light.

We now put a quarter wave plates on the path of the beam followed by the rotating Polaroid. If there is no variation of intensity, then the incident beam is unpolarized. If there is complete extinction at two positions, then the beam is circularly polarized (this is so because a quarter wave plate will transform a circularly polarized light into a linearly polarized light). If there is a variation of intensity (without complete extinction), then the beam is a mixture of unpolarized and circularly polarized light.

3. If there is a variation of intensity (without complete extinction), then the beam is elliptically polarized or a mixture of linearly polarized and unpolarized or a mixture of elliptically polarized and unpolarized light. We now put a quarter wave plate in front of the Polaroid with its optic axis parallel to the pass axis of the Polaroid at the position of maximum intensity. The elliptically Polarized light will transform to a linearly polarized light. Thus, if one obtains two positions of the Polaroid where complete extinction occurs, then the original beam is elliptically polarized. If complete extinction does not occur and the position of maximum intensity occurs at the same orientation as before, the beam is a mixture of unpolarized and linearly polarized light. Finally, if the position of maximum intensity occurs at a different orientation of the Polaroid, the beam is a mixture of elliptically polarized and unpolarized light.

MCQ (Multiple Choice Questions)

Q1. A beam of unpolarized light of intensity I_0 passes through a combination of an ideal polarizer and an idea analyzer with their transmission axes at 60^o. What is the intensity of the beam coming out at the other end?

(a) I_0 (b) $\frac{I_0}{2}$ (c) $\frac{I_0}{4}$ (d) $\frac{I_0}{8}$

Q2. A quarter wave plate is placed normally in the path of a plane polarized light beam. The angle between the electric vector and the fast-axis of the plate is 250. The beam coming out of the quarter wave plate will be which one of the following ?

(a) Plane polarized (b) Circularly polarized
(c) Elliptically polarized (d) Unpolarized

Q3. When light passing through rotating Nicol prism is observed, no change in intensity is seen. What inference can be drawn?

(a) The incident light is unpolarized.
(b) The incident light is circularly polarized.
(c) The incident light is uppolarized or circularly polarized.
(d) The incident light is uppolarized or circularly polarized or combination of both.

Q4. When light travels along the optic axis of quartz, which one of the following statements is correct regarding occurrence of optical activity and double refraction?

(a) only optical activity is observed
(b) only double reflection is observed
(c) Both are observed and are at their maximum
(d) None of the two is observed along the optic axis

Q5. A quarter wave plate is placed over a shiny coin. A plane polarizer is placed on top of the quarter wave plate such that the transmission axis of the polarizer is at 45^o to the fast axis of the quarter wave plate. How does the shiny coin appear now?

(a) Dark (b) Shiny as before
(c) Shinier than before (d) Coloured

Q6. A circularly polarized beam of light passes through a quarter wave place. The emerging beam is:

(a) plane polarized (b) circularly polarised

(c) elliptically polarized (d) partially polarized

Q7. Which one of the following statements is correct?

Optically active substances are responsible for:

(a) the rotation of the plane of polarization polarized light

(b) producing polarized light

(c) producing brifrigence

(d) converting ordinary light into polarized light

Q8. Match List – I (Phase Difference between Two Similar Superimposed Waves having Mutually Perpendicular States of Polarization and Propagating through the Same Axis) with List – II (Result) and select the correct answer using the code given below the Lists:

List I (Phase Difference between two similar superimposed waves having mutually perpendicular states of polarization and propagating through the same axis)	**List II** **(Result)**
A. $\theta = 0$	1. Linearly polarized light
B. $\theta = \dfrac{\pi}{2}$	2. Left circularly polarized light
C. $\theta = \dfrac{2\pi}{3}$	3. Right circularly polarized light
D. $\theta = \dfrac{3\pi}{2}$	4. Elliptically polarized light.

Codes:

	A	B	C	D
(a)	1	3	4	2
(b)	4	2	1	3
(c)	1	2	4	3
(d)	4	3	1	2

Q9. Consider two light waves represented by two mutually perpendicular electric field vectors: $E_x = A_x \cos(\omega_x t + \phi_x)$ and $E_y = A_y \sin(\omega_y t + \phi_y)$. Their superposition will result in a plane polarized light, if:

(a) $A_x = A_y, \phi_x = \dfrac{\pi}{2}, \phi_y = \pi$ (b) $\omega_x = \omega_y, \phi_x = \dfrac{\pi}{2}, \phi_y = \pi$

(c) $A_x \neq A_y, \omega_x = \omega_y, \phi_x = \phi_y = 0$ (d) $A_x = A_y, \omega_x = \omega_y, \phi_x \neq \phi_y$

Q10. The electric field components of a plane electromagnetic wave are

$E_x = 2E_0 \cos(\omega t - kz); E_y = E_0 \sin(\omega t - kz)$

The state of polarization of the wave will be:

(a) circular (b) plane (c) elliptical (d) unpolarized

Q11. Match List I with List II and select the correct answer using the codes given below

List I	**List II**
A. Double refraction	1. Wollaston
B. Polarizing or Analyzing prism	2. Babinet
C. Double image prism	3. Quartz crystal
D. Elliptically polarized light	4. Polarimeter

Codes:

	A	B	C	D
(a)	3	5	4	2
(b)	2	5	4	3
(c)	3	4	1	2
(d)	2	4	1	3

Q12. When monochromatic light is passed through an analyzing nicol prism, the intensity of the emergent light is found to very between a maximum and a non-zero minimum. Which one of the following cannot represent the polarization characteristics of the incident light?

(a) Mixture of plane-polarised and unpolarised light

(b) Mixture of plane-polarised and circularly-polarised light

(c) Elliptically-polarised light

(d) Plane-polarised light

Q13. A given calcite plate behaves as a half-wave plate for a particular wavelength λ. If the variation of refractive index with λ is negligible, then for a light of wavelength 2λ, the given plate would behave as a:

(a) half-wave-plate (b) quarter-wave plate

(c) plane Polaroid (d) non-polarizing plate

Q14. Two polarizing sheets have their polarizing directions parallel, so that the intensity of the transmitted light is maximum. If the intensity is to drop by one-half, then either of the two sheets must be turned by:

(a) $\pm 30^o$ and $\pm 135^o$ (b) $\pm 45^o$ and $\pm 120^o$

(c) $\pm 30^o$ and $\pm 120^o$ (d) $\pm 45^o$ and $\pm 135^o$

Q15. If t is the minimum thickness of a quarter wave plate needed to convert plane polarized light of wavelength $480\ nm$ into circular polarized light, then the corresponding thickness of a quarter wave plate for wavelength $600\ nm$ is:

(a) $0.56t$ (b) $0.75\ t$ (c) $1.25\ t$ (d) $1.44\ t$

Q16. The super position of two plane polarized lights in two mutually perpendicular directions given by

$$x = 4\sin\left(\omega t + \frac{\pi}{4}\right) \text{ and } y = 5\sin\left(\omega t + \frac{3}{4}\pi\right)$$

will result in:

(a) plane polarized light (b) unpolarized light

(c) elliptically polarized light (d) circularly polarized light

Q17. If a quarter wave-plate with its fast axis parallel to the surface is inserted into a beam of linearly polarized light oscillating at 45^o with fast axis, then the emerging light will be :

(a) linearly polarized (b) vertically polarized

(c) left circularly polarized (d) left elliptically polarized

Q18. The thickness of quarter-wave plate made from a doubly refracting crystal is $6.7\times10^{-5}\ cm$ for a light of wavelength $4800\overset{0}{A}$. What is the corresponding thickness of half-wave plate for a light of wavelength $6000\overset{0}{A}$?

(a) $16.75\times10^{-5}\ cm$ (b) $13.4\times10^{-5}\ cm$

(c) $10.72\times10^{-5}\ cm$ (d) $8.34\times10^{-5}\ cm$

Q19. A mixture of unpolarized and circularly polarized light is passed through a quarter wave plate and then through an analyzer is rotated:

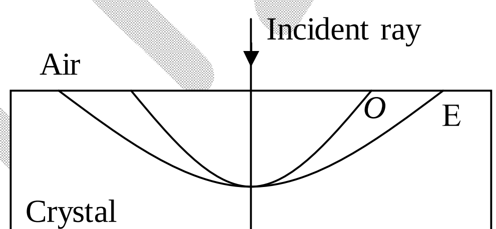
(a) there will be no variation in intensity

(b) there are maxima and minima in intensity with $I_{\min}=0$

(c) there are maxima and minima in intensity with $I_{\min}\neq0$

(d) the result depends on the orientation of the quarter wave plate

Q20. The given figure illustrates the passage of the ordinary and extra ordinary wavefronts O and E through a uniaxial crystal. The figure represents crystal orientation with its:



(a) optic axis in the plane of incidence and parallel to the crystal surface

(b) optic axis perpendicular to the plane of incidence and parallel to the crystal surface

(c) optic axis in the plane of incidence and perpendicular to the crystal surface

(d) optic axis perpendicular to the plane of incidence and perpendicular to the crystal surface

Q21. An incident light is viewed through a rotating nicol prism. Match List I with List II and select the correct answer by using the codes given below the lists:

List I (Polarization of light incident of rotating nicol prism)	**List II** (Intensity of light as observed through rotating nicol prism)
A. Circularly polarized	1. Intensity changes and vanishes
B. Elliptically polarized	2. Intensity changes but is never completely extinguished
C. Mixture of plane polarized and unpolarized light	3. Intensity does not change
D. Plane polarized light	

Codes:

	A	B	C	D	
(a)	2	3	1	2	3
(b)	1	1	2	3	3
(c)	3	2	2	1	3
(d)	3	2	1	3	1

Q22. A uniaxial birefringent crystal is cut to form a parallel plate with its optic axis parallel to the front face. For such a retarder plate of refractive indices n_0 and n_e, with thickness d, the expression for the relative phase difference, $\Delta\phi$, for the wavelength λ is given by:

(a) $\dfrac{|n_0 - n_e|d}{\lambda}$

(b) $\dfrac{\pi|n_0 - n_e|d}{\lambda}$

(c) $\dfrac{\pi|n_0 - n_e|d}{2\lambda}$

(d) $\dfrac{2\pi|n_0 - n_e|d}{\lambda}$

Q23. Group I contains x - and y - components of the electric field and Group II contains the type of polarization of light.

	Group I	**Group II**
P.	$E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$ $E_y = E_0 \sin(\omega t + kz + \pi)$	1. Circularly Polarized
Q.	$E_x = E_0 \sin\left(\omega t + kz + \frac{\pi}{6}\right)$ $E_y = E_0 \sin\left(\omega t + kz - \frac{\pi}{3}\right)$	2. Elliptically Polarized
R.	$E_x = E_0 \sin\left(\omega t + kz + \frac{3\pi}{4}\right)$ $E_y = E_0 \sin\left(\omega t + kz - \frac{\pi}{4}\right)$	3. Linearly Polarized

The correct set of matches is

(a) $P \rightarrow 1; Q \rightarrow 2; R \rightarrow 3$ (b) $P \rightarrow 1; Q \rightarrow 3; R \rightarrow 2$

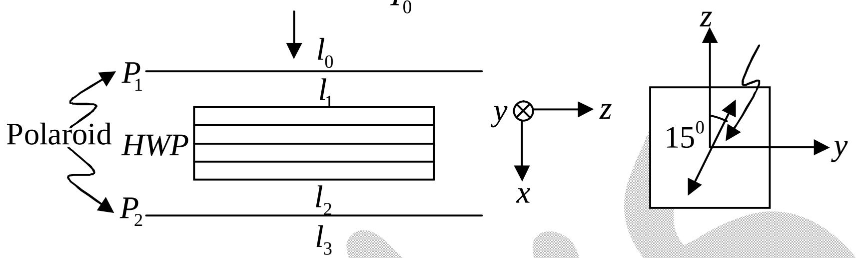
(c) $P \rightarrow 2; Q \rightarrow 1; R \rightarrow 3$ (d) $P \rightarrow 2; Q \rightarrow 3; R \rightarrow 1$

Q24. The thickness of a quarter wave plate made of quartz for wavelength $\lambda = 6000\,\overset{0}{A}$, refractive indices $\mu_e = 1.553$ and $\mu_0 = 1.543$ is:

(a) 1.25×10^{-3} cm (b) 0.15×10^{-3} cm

(c) 1.50×10^{-3} cm (d) 2.50×10^{-3} cm

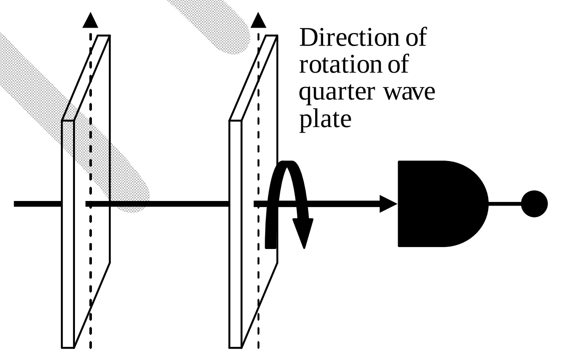
Q25. A HWP is introduced between two crossed Polaroids P_1 and P_2. The optic axis makes an angle of 15^o with the pass axis of P_1 as shown in Fig. (a) and (b). If an unpolarized beam of intensity I_0 is normally incident on P_1 and if I_1, I_2, and I_3 are the intensities after P_1, after HWP, and after P_2, respectively, $\frac{I_3}{I_0}$ is



(a) (b)

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{8}$

Q26. A half-wave plate is placed in between a polarizer and a photo-director. When the optic axis of the quarter-wave plate is kept initially parallel to the pass axis of the polarizer and perpendicular to the direction of light propagation. The intensity of light passing through the quarter-wave plate is measured to be I_0 (see figure). If the quarter wave plate is now rotated by 45^0 about an axis parallel to the light propagation, what would be the intensity of the emergent light measured by the photo-director?



(a) $\frac{I_o}{\sqrt{2}}$ (b) $\frac{I_0}{2}$ (c) $\frac{I_0}{2\sqrt{2}}$ (d) I_0

Q27. Unpolarized light of intensity I pass through two Polaroid sheets. The axis of the first is vertical, and that of the second is at 30^0, to the vertical, what fraction of the incident light is transmitted?

(a) $\frac{3}{4}I$ (b) $\frac{1}{2}I$ (c) $\frac{1}{4}I$ (d) $\frac{3}{8}I$

MSQ (Multiple Select Questions)

Q29. A beam of light passes through a quarter wave plate and the emergent beam is circularly polarized. For this to happen,

(a) electric vector should be parallel to the optic axis

(b) incident beam should be plane polarized

(c) incident beam should be unpolarized

(d) electric vector should make an angle of 45^o with the optic axis

Q30. Consider the following statements:

A beam of light passes through a quarter wave plate and the emergent beam is circularly polarized. For this to happen,

(a). incident beam should be plane polarized

(b). electric vector should make an angle of 45^o with the fast axis

(c). incident beam should be unpolarized

(d). electric vector should be parallel to the fast axis

Q31. Consider the following statements:

When plane polarized monochromatic light passes through an optically active substance,

(a). its plane of polarization rotates clockwise for all substances.

(b). its plane of polarization rotates anticlockwise for all substances.

(c). the angle of rotation of plane of polarization is proportional to the distance traveled inside the substance.

(d) the angle of rotation of plane of polarization is inversely proportional to the distance traveled inside the substance.

Q32. When an unpolarised light beam passes through a double refracting medium, it splits up into two beams called ordinary ray and extraordinary ray.

Consider the following statements.

(a). Intensities of the both rays are equal.

(b). Refractive index of the ordinary ray remains constant.

(c). Refractive index of the extraordinary ray does not remain constant.

(d). Both the rays are polarized.

Q33. Group I contains x- and y- components of the electric field and Group II contains the type of polarization of light.

(1) $E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$, $E_y = E_0 \sin(\omega t + kz)$

(2) $E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \cos(\omega t + kz)$

(3) $E_x = E_1 \sin(\omega t + kz)$, $E_y = E_2 \sin(\omega t + kz)$

(4) $E_x = E_0 \sin(\omega t + kz)$, $E_y = E_0 \sin\left(\omega t + kz + \frac{\pi}{4}\right)$

Which of the following statements are correct

(a) Resultant of (1) leads to elliptically polarized

(b) Resultant of (2) leads to circularly polarized

(c) Resultant of (3) leads to linarly polarized

(d) Resultant of (4) leads to Left circularly polarized

NAT (Numerical Answer Type)

Q34. Unpolarised light of intensity I_0 gets polarized emerges from a polarizer. It then passes through an analyzer. The intensity of light emerging from the analyzer is $\frac{I_0}{8}$. The angle between the polarizing direction of the polarizer and analyzer is ……………. degree

Q35. A quarter wave plate is designed for a wavelength of $600\,nm$. The difference in refractive indices for the electric components along the fast and the slow axes is 0.2. The geometrical thickness of the plate will be ………. mm

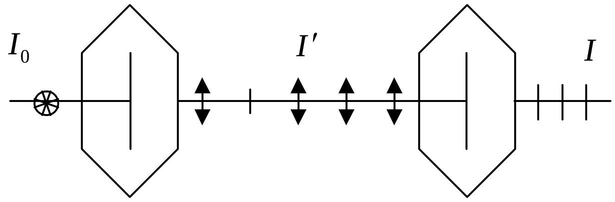
Q36. Unpolarized light in incident on a glass plate having refractive index 1.5. The angle of incidence at which the plane polarized light is obtained, is ……………….. degree

Q37. The thickness of a quarter wave plate made of quartz for wavelength $\lambda = 5000\,\overset{0}{A}$, refractive indices $\mu_e = 1.553$ and $\mu_0 = 1.543$ is__________ $\times 10^{-3}\,cm$.

Solution-MCQ (Multiple Choice Questions)

Ans. 1: (d)

Solution:



When an unploarized of intensity I_0 passes through a polarizer then intensity becomes half i.e., $I' = \frac{1}{2} I_0$ -------(i)

Now by Maulus law $I = I'\cos^2\theta$ where $\theta = 60^0$

$$\Rightarrow \quad I = \frac{I_0}{2}\cos^2 60^0 = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

Ans. 2: (c)

Solution: The quarter wave plate is a transparent plate which produces a path difference of $\frac{\lambda}{4}$ for ordinary and extraordinary ray.

If the polarized light makes an angle of 25^o with optic axis then amplitude of extraordinary ray is $E_0 \cos\phi$ and ordinary ray is $E_0 \sin\phi$

Since, their amplitudes are same hence, they from elliptical polarized light after emerging from the plate.

Ans. 3: (d)

Ans. 4: (a)

Solution: The rotation of plane of vibration when a polarized light passes through an optical active medium is called optical activity.

Ans. 5: (d)

Solution: A quarter wave plate kept at an angle of 45^0 with optic axis produces plane polarized light hence, coin appears colored.

Ans. 6: (a)

Solution:

$$CP \xrightarrow{\delta_1 = \pi/2} \overset{QWP}{\boxed{\delta = \pi/2}} \longrightarrow LP$$

When a circularly polarized beam passes through a quarter wave plate, it becomes plane polarized.

Because the quarter wave plate is plate which can produces the path difference of $\frac{\lambda}{4}$ and phase difference of $\frac{\pi}{2}$ between ordinary and extra ordinary ray.

The total phase difference after QWP is $\delta = \delta_1 + \delta_2 = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

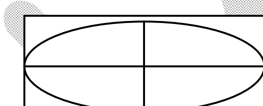
Thus output light will be linearly polarized

Ans. 7: (a)

Solution: When a plane polarized light enters in an optically active substance the plane polarized light is rotated clockwise or anticlockwise depending on nature of the substance.

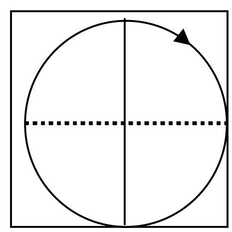
Ans. 8: (a)

Solution:



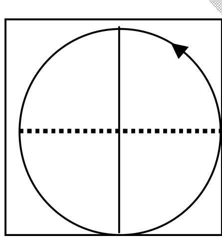
elliptical

$\phi = \frac{2\pi}{3}$



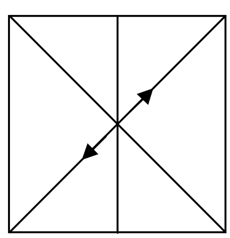
left circular

$\phi = \frac{2\pi}{3}$



right circular

$\theta = \frac{\pi}{2}$



plane polarised

$\theta = 0$

Ans. 9: (b)

Solution: Te ordinary light is a wave in which its both components E and B vibrate perpendicularly to the direction of the light. If E vibrate in a plane the light is said to plane polarized light.

The two perpendicular vibrating light waves from a plane polarized light if their frequencies are equal $\omega_x = \omega_y$ and have a phase difference of zero. .

$$E_x = A_x \cos\left(\omega_x t + \phi_x\right) = A_x \sin\left(\omega_x t + \phi_x + \frac{\pi}{2}\right) \text{and} \qquad E_y = A_y \sin\left(\omega_y t + \phi_y\right)$$

The phase difference is $\delta = \phi_x + \frac{\pi}{2} - \phi_y = 0 \quad \Rightarrow \phi_x - \phi_y = -\frac{\pi}{2}$

This is possible when $\phi_x = \frac{\pi}{2} + \phi_y = \pi$.

Ans. 10: (c)

Solution: We have $E_x = 2E_0 \cos(\omega t - kz)$

$$\Rightarrow \qquad \frac{E_x}{2E_0} = \cos(\omega t - kz) \qquad \text{-------(i)}$$

and $\quad E_y = E_0 \sin\left(\omega t - kz\right)$

$$\Rightarrow \qquad \frac{E_y}{E_0} = \sin\left(\omega t - kz\right) \qquad \text{--------(ii)}$$

squaring equations. (i) and (ii) and adding, we get

$$\frac{E_x^2}{4E_0^2} + \frac{E_y^2}{E_0^2} = 1$$

This represents equation of ellipse.

Hence, it is elliptically polarized.

Ans. 11: (c)

Ans. 12: (d)

Solution: When a plane polarized light passes through an analyzer and analyzer, is rotating then intensity becomes maximum and a zero minimum twice for one complete rotation of the analyzer.

Ans. 13: (b)

Solution: A plate which produces a path difference of $\left(\frac{\lambda}{4}\right)$ between ordinary and extraordinary rays is called quarter-wave plate. If this path difference is $\left(\frac{\lambda}{2}\right)$, then it is called Half-wave plate.

For quarter-wave plate $t_{1/4} = \dfrac{\lambda}{4(\mu_e - \mu_0)}$

For half-wave plate $t_{1/2} = \dfrac{\lambda}{2(\mu_e - \mu_0)}$

If t is width of plate then $(\mu_0 - \mu_e)t = \left(\frac{\lambda}{2}\right)$

Again, $(\mu_0 - \mu_e)t = \dfrac{2\lambda}{4}$

Hence, for wavelength 2λ the plate will behave as a quarter-wave plate.

Ans. 14: (d)

Solution: According to Malaus law intensity is proportional the square of the cosine of angle between and polarizer, i.e. $I \propto \cos^2\theta$ Here $\cos^2\theta = \dfrac{1}{2}$

$\Rightarrow \cos\theta = \pm\dfrac{1}{\sqrt{2}} \Rightarrow \theta = \pm 45^\circ, \pm 135^\circ$.

Ans. 15: (c)

Solution: If λ is wavelength of a monochromatic light thickness of the quarter wavelength is as

Let $t_1 = \dfrac{\lambda_1}{4(\mu_0 - \mu_c)}$ and $t_2 = \dfrac{\lambda_2}{4(\mu_0 - \mu_c)}$

Thus, $t_2 = t_1\dfrac{\lambda_2}{\lambda_1} = t_1\dfrac{600}{480} = t\left(\dfrac{5}{4}\right) = 1.25\, t$

Ans. 16: (c)

Solution: Let $y = a\sin(\omega t + \phi)$ and $x = b\sin\omega t$

Given $x = 4\sin\left(\omega t + \frac{\pi}{4}\right)$ and $y = 5\sin\left(\omega t + \frac{3\pi}{4}\right)$

where, $a = 5, b = 4,$ and $\phi = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$

Since $a \neq b$ and $\phi = \frac{\pi}{2}$, so resultant will be elliptically polarized.

Ans. 17: (c)

Solution: As the surface of QWP, the electric field components are

$$E_x = (E_0\cos\theta)\sin(\omega t)\,\hat{x} = \frac{E_0}{\sqrt{2}}\sin\omega t\,\hat{x}$$

$$E_y = (E_0\cos\theta)\sin(\omega t)\,\hat{y} = \frac{E_0}{\sqrt{2}}\sin\omega t\,\hat{y}$$

Thus at the output of QWP, the E - field components are

$$E_x = \frac{E_0}{\sqrt{2}}\sin(\omega t) \text{ and } E_y = \frac{E_0}{\sqrt{2}}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Thus the output will be left circularly polarized light.

The quarter wave plate for which the path difference between ordinary ray and extra ordinary ray is $\frac{\lambda}{4}$ or phase difference is $\left(\frac{\pi}{2}\right)$.

Ans. 18: (a)

Solution: If n_e and n_0 are refractive indices of extraordinary and ordinary rays in case of doubly refraction then thickness of wave plate are given as

$(n_e - n_0) =$ (in term of wavelength)

For half wavelength $t_1 = \frac{\lambda}{2}$ --------(i)

For quarter wavelength $t_2 = \frac{\lambda}{4}$ --------(ii)

By equation (i) and (ii) half wave plate $=2\times$ thickness of quarter wave plate

So, $$\left(n_e-n_0\right)\frac{t_1}{4}=\frac{\lambda'}{4} \quad \text{--------(iii)}$$

and $$\left(n_e-n_0\right)t_{1/2}=\frac{\lambda''}{2} \quad \text{--------(iv)}$$

Dividing equation (iv) by (iii), we get

$$t_{1/2}=\left(\frac{\lambda''}{2}\right)\left(\frac{4}{\lambda'}\right)\cdot t_{1/4}$$

$$\Rightarrow \quad t_{1/2}=\left(\frac{6000}{2}\right)\left(\frac{4}{4800}\right)\times 6.7\times 10^{-5}=16.75\times 10^{-5}\ cm$$

Ans. 19: (c)

Solution: When unpolarized with circular polarized light is passed through a quarter wave plate, it becomes a plane polarized light and hence if it again passes through an analyzer the intensity becomes maximum and minimum due to the rotation of analyzer but minima never become zero.

Ans. 20: (d)

Solution: Extra ordinary ray travels with different velocities whereas the ordinary ray travels with same velocity in all directions. The velocities of extra ordinary and ordinary rays are same along the optic axis. From the figure, it is clear that along the incident ray, their velocities are equal. Hence, the optic axis is along the incident ray.

Ans. 21: (c)

Solution:

(i) Circularly polarized light	Intensity does not change
(ii) Unpolarised light	Intensity does not change
(iii) Elliptically polarised	Intensity changes but never completely extinguished
(iv) Mixture of plane polarized	Intensity changes but never completely extinguished
(v) Plane polarized light	Intensity changes and vanishes

Ans. 22: (d)

Solution: The path difference $\delta = (n_0 - n_e)d$

$\Rightarrow$ The phase difference $\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi|n_0 - n_e|d}{\lambda}$

Ans. 23: (c)

P. $E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$ *and* $E_y = E_0 \sin(\omega t + kz + \pi)$

The phase difference between E_x and E_y is $\frac{\pi}{2}$ with different amplitude. Therefore the resultant will be elliptically polarized.

Q. $E_x = E_0 \sin\left(\omega t + kz + \frac{\pi}{6}\right)$ *and* $E_y = E_0 \sin\left(\omega t + kz - \frac{\pi}{3}\right)$

The phase difference between E_x and E_y is $\frac{\pi}{2}$ with same amplitude. Therefore the resultant will be circularly polarized.

R. $E_x = E_1 \sin\left(\omega t + kz + \frac{3\pi}{4}\right)$ *and* $E_y = E_2 \sin\left(\omega t + kz - \frac{\pi}{4}\right)$

The phase difference between E_x and E_y is π with different amplitude. Therefore the resultant will be linarly polarized.

Ans. 24: (c)

Solution: The plate which produce a path difference of $\left(\frac{\lambda}{4}\right)$ for extraordinary and ordinary rays is called quarter wave plate.

Hence , $(\mu_e - \mu_0)t = \frac{\lambda}{4} \Rightarrow$ $(1.533 - 1.543)t = \frac{6000 \times 10^{-10}}{4} m$

$\Rightarrow$ $0.010t = \frac{6000 \times 10^{-10}}{4} m$ $\Rightarrow$ $t = 1.5 \times 10^{-3} cm$

Ans. 25: (d)

Solution: Unpolarized light of Intensity I_0 incident on Polaroid P_1, the intensity I_1 will be $\frac{I_0}{2}$.

HWP introduce a phase difference of π, output of HWP will be linearly polarized with no change in the intensity at the output of HWP. The electric field vector of linearly polarized light will make angle 30° with the pass axis of the second polarizer.

According to Malu's law intensity at output will be

$$I_3 = \frac{I_0}{2}\cos^2\theta = \frac{I_0}{2}\cos^2(\pi/6) = \frac{I_0}{8}$$

Ans. 26: (d)

Solution: After passing through half wave plate the plane polarized light of intensity I_0 will remain in early polarized rotated at 45^0 but the intensity is driven by Malu's law

$$I = I_0\cos^2(\theta) = I_0\cos^2(45) = \frac{I_0}{2}$$

Ans. 27: (d)

Solution: The average value of the cosine squard function is one-half, so the first polarizer transmits $\frac{1}{2}I$ of the light.

The second transmits $\cos^2\left(30^0\right) = \frac{3}{4}$

$$\therefore \quad I_f = \frac{1}{2}\times\frac{3}{4}I = \frac{3}{8}I$$

MSQ (Multiple Select Questions)

Ans. 29: (b) and (d)

Ans. 30: (a) and (b)

Solution: A transparent plate which can produce a path difference of one quarter of wavelength i.e., $\frac{\lambda}{4}$ between extraordinary and ordinary ray is called quarter wave plate.

For this to happen

(i) incident beam should be plane polarized

(ii) electric vector E should make an angle of 45° with the fast axis

Ans. 31: (d)

Solution: When a plane polarized light passes through an special type of substance along the optic axis the plane of polarization is rotated clockwise or anticlockwise depending on the nature of the substance. This special type of substance is called optically active substance and this phenomenon is known as optic activity.

Ans. 32: (b) and (c)

Solution: when an unpolarised light beam passes through a double refracting medium it splits into two beams viz extraordinary ray which has different velocity in different directions and other is ordinary ray whose velocity in all directions is same.

Both these are plane polarized and the plate of polarizations of these are perpendicular to each other.

Ans. 33: (a), (b) and (c)

Solutions:

1. $E_x = \frac{E_0}{\sqrt{2}}\cos(\omega t + kz)$ *and* $E_y = E_0 \sin(\omega t + kz)$

The phase difference between E_x and E_y is $\frac{\pi}{2}$ with different amplitude. Therefore the resultant will be elliptically polarized.

2. $E_x = E_0 \sin(\omega t + kz)$ *and* $E_y = E_0 \cos(\omega t + kz)$

The phase difference between E_x and E_y is $\frac{\pi}{2}$ with same amplitude. Therefore the resultant will be circularly polarized.

3. $E_x = E_1 \sin(\omega t + kz)$ *and* $E_y = E_2 \sin(\omega t + kz)$

The phase difference between E_x and E_y is 0 with different amplitude. Therefore the resultant will be linarly polarized.

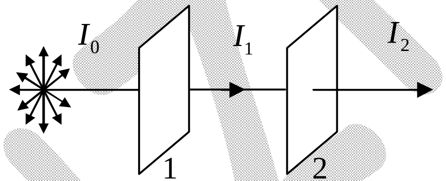
4. $E_x = E_0 \sin(\omega t + kz)$ *and* $E_y = E_0 \sin\left(\omega t + kz + \frac{\pi}{4}\right)$

The phase difference between E_x and E_y is $\frac{\pi}{4}$ with same amplitude. Therefore the resultant will be elliptically polarized.

NAT (Numerical Answer Type)

Ans. 34: 60^o

Solution:



$2 \rightarrow$ Analyzer

$1 \rightarrow$ Polarizer

Since, average of $\cos^2\theta$ is $\frac{1}{2}$ so if I_0 is intensity of unpolarised light then intensity of polarized coming out from the polarizer is $\frac{1}{2}$

Thus, $I_1 = \frac{1}{2} I_0$ ------(i)

By Malaus law, the intensity of light coming out from the analyzer

$$I_2 = I_p \cos^2\theta \quad \text{------(ii)}$$

By equation (i) & (ii), we get

$$I_2 = \frac{1}{2} I_0 \cos^2\theta \qquad \text{------(iii)}$$

By question $$I_2 = \frac{1}{8} I_0 \qquad \text{-------(iv)}$$

By equation (iii) and (iv) $\frac{1}{8} I_0 = \frac{1}{2} I_0 \cos^2\theta$

$$\Rightarrow \quad \cos^2\theta = \frac{1}{4} \quad \Rightarrow \quad \cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60^0$$

Ans. 35: 750

Solution: A quarter wave plate is plate for which the difference of path between extra ordinary and ordinary ray is $\frac{\lambda}{4}$.

$$(\mu_e - \mu_o)t = \frac{\lambda}{4}$$

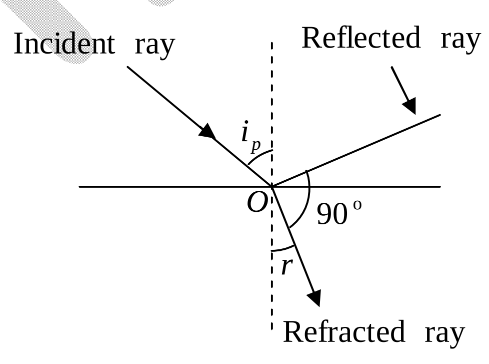
Hence, $\mu_e - \mu_o = 0.2, \lambda = 600 \times 10^{-9}$

So, $$t = \frac{\lambda}{4(\mu_e - \mu_o)}$$

$$\Rightarrow \quad t = \frac{600 \times 10^{-9}}{4(0.2)} m = 750\ nm$$

Ans. 36: 57

Solution:



By Brewster's law:

$$\tan i_p = n$$

$$\Rightarrow \quad \tan i_p = 1.5 \quad \text{(given)}$$

$$\Rightarrow \quad i_p = 57^0$$

Ans. 37: 1.25

Solution: The plate which produce a path difference of $\left(\frac{\lambda}{4}\right)$ for extraordinary and ordinary rays is called quarter wave plate.

Hence , $(\mu_e - \mu_0)t = \frac{\lambda}{4}$

$\Rightarrow \quad (1.533 - 1.543)t = \frac{5000 \times 10^{-10}}{4} m$

$\Rightarrow \quad 0.010t = \frac{5000 \times 10^{-10}}{4} m$

$\Rightarrow \quad t = 1.25 \times 10^{-3} cm$