

## 1. Semiconductor Physics

### 1.1 Metals, Semiconductors, and Insulators

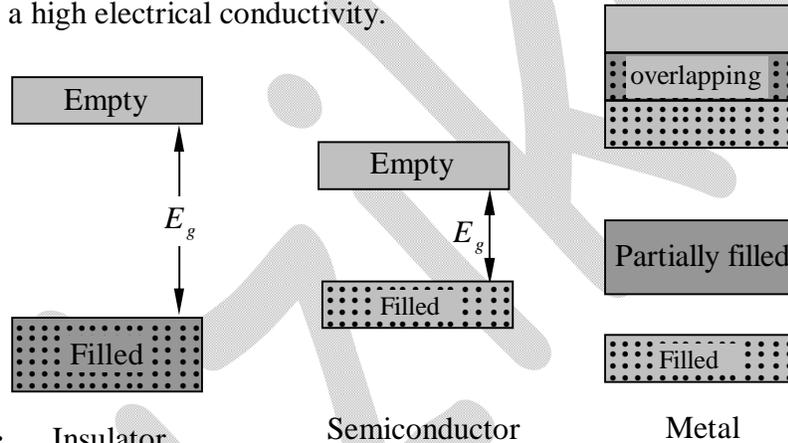
Every solid has its own characteristic energy band structure. This variation in band structure is responsible for the wide range of electrical characteristics observed in various materials. The diamond band structure for example, can give a good picture of why carbon in the diamond lattice is a good insulator. To reach such a conclusion, we must consider the properties of completely filled and completely empty energy bands in the current conduction process.

Before discussing the mechanisms of current flow in solids further, we can observe here that for electrons to experience acceleration in an applied electric field, they must be able to move into new energy states. This implies there must be empty states (allowed energy states which are not already occupied by electrons) available to the electrons. For example, if relatively few electrons reside in an otherwise empty band, ample unoccupied states are available into which the electrons can move. On the other hand, the diamond structure is such that the valence band is completely filled with electrons at  $0K$  and the conduction band is empty. There can be no charge transport within the valence band, since no empty states are available into which electrons can move. There are no electrons in the conduction band, so no charge transport can take place there either. Thus carbon in the diamond structure has a high resistivity typical of insulators.

**Semiconductor materials** at  $0K$  have basically the same structure as insulators—a filled valence band separated from an empty conduction band by a band gap containing no allowed energy states (Figure 1.1). The difference lies in the size of the band gap  $E_g$  which is much smaller in semiconductors than in insulators. For example, the semiconductor *Si* has a band gap of about  $1.1 eV$  compared with  $5 eV$  for diamond. The relatively small band gaps of semiconductors allow for excitation of electrons from the lower (valence) band to the upper (conduction) band by reasonable amounts of thermal or optical energy.

For example, at room temperature a semiconductor with a  $1\text{ eV}$  band gap will have a significant number of electrons excited thermally across the energy gap into the conduction band whereas an insulator with  $E_g = 10\text{ eV}$  will have a negligible number of such excitations. Thus an important difference between semiconductors and insulators is that the number of electrons available for conduction can be increased greatly in semiconductors by thermal or optical energy.

In metals the bands either overlap or are only partially filled. Thus electrons and empty energy states are intermixed within the bands so that electrons can move freely under the influence of an electric field. As expected from the metallic band structures, metals have a high electrical conductivity.



**Figure 1.1:** Insulator                      Semiconductor                      Metal

## 1.2 Direct and Indirect Semiconductors

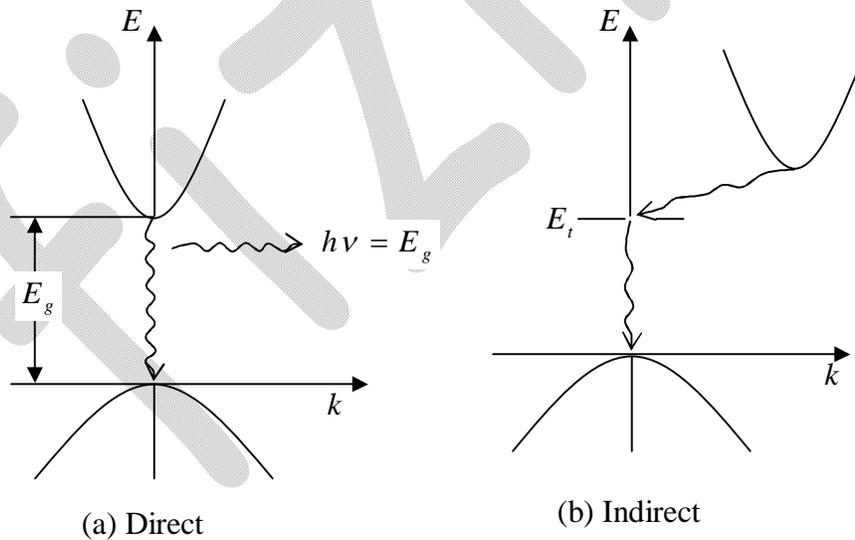
When quantitative calculations are made of band structures, a single electron is assumed to travel through a perfectly periodic lattice. The wave function of the electron is assumed to be in the form of a plane wave moving, for example, in the  $x$ -direction with propagation constant  $k$ , also called a wave vector. The space-dependent wave function for the electron is

$$\psi_k(x) = U(k_x, x)e^{ik_x x}$$

where the function  $U(k_x, x)$  modulates the wave function according to the periodicity of the lattice.

In such a calculation, allowed values of energy can be plotted vs. the propagation constant  $k$ . Since the periodicity of most lattices is different in various directions, the  $(E, k)$  diagram must be plotted for the various crystal directions and the full relationship between  $E$  and  $k$  is a complex surface which should be visualized in three dimensions.

The band structure of  $GaAs$  has a minimum in the conduction band and a maximum in the valence band for the same  $k$  value ( $k=0$ ). On the other hand,  $Si$  has its valence band maximum at a different value of  $k$  than its conduction band minimum. Thus an electron making a smallest-energy transition from the conduction band to the valence band in  $GaAs$  can do so without a change in  $k$  value; on the other hand a transition from the minimum point in the  $Si$  conduction band to the maximum point of the valence band requires some change in  $k$ . Thus there are two classes of semiconductor energy bands *direct* and *indirect* (Figure 1.2). We can show that an indirect transition involving a change in  $k$  requires a change of momentum for the electron.



**Figure 1.2:** Direct and indirect electron transitions in semiconductors: (a) direct transition with accompanying photon emission; (b) indirect transition via a defect level.

In a direct semiconductor such as  $GaAs$ , an electron in the conduction band can fall to an empty state in the valence band, giving off the energy difference  $E_g$  as a photon of light. On the other hand, an electron in the conduction band minimum of an indirect semiconductor such as  $Si$  cannot fall directly to the valence band maximum but must undergo a momentum change as well as changing its energy. For example, it may go through some defect state ( $E_i$ ) within the band gap. In an indirect transition which involves a change in  $k$ , the energy is generally given up as heat to the lattice rather than as an emitted photon. This difference between direct and indirect band structures is very important for deciding which semiconductors can be used in devices requiring light output. For example, semiconductor light emitters and lasers generally must be made of materials capable of direct band-to-band transitions or of indirect materials with vertical transitions between defect states.

### 1.3 Electrons and Holes

As the temperature of a semiconductor is raised from  $0 K$ , some electrons in the valence band receive enough thermal energy to be excited across the band gap to the conduction band. The result is a material with some electrons in an otherwise empty conduction band and some unoccupied states in an otherwise filled valence band (Figure 1.3). For convenience, an empty state in the valence band is referred to as a *hole*. If the conduction band electron and the hole are created by the excitation of a valence band electron to the conduction band, they are called an *electron-hole pair* (abbreviated EHP).

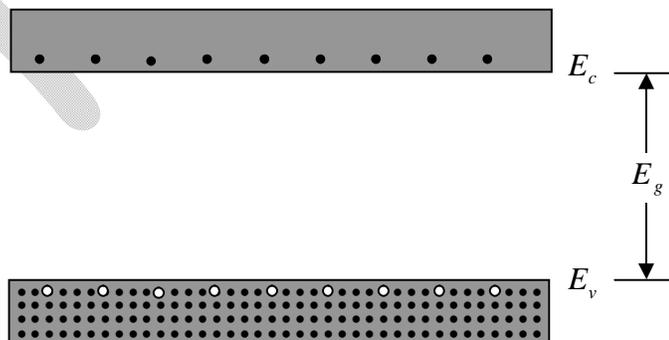


Figure 1.3: Electron-hole pairs in a semiconductor.

After excitation to the conduction band, an electron is surrounded by a large number of unoccupied energy states. For example, the equilibrium number of electron-hole pairs in pure *Si* at room temperature is only about  $10^{10} \text{ EHP/cm}^3$ , compared to the *Si* atom density of more than  $10^{22} \text{ atoms/cm}^3$ . Thus the few electrons in the conduction band are free to move about via the many available empty states.

### 1.3.1 Effective Mass

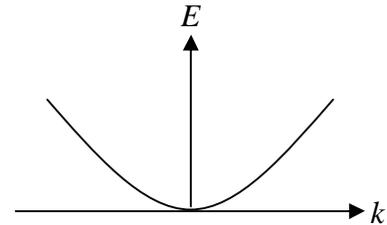
The electrons in a crystal are not completely free, but instead interact with the periodic potential of the lattice. As a result, their “wave-particle” motion can-not be expected to be the same as for electrons in free space. Thus, in applying the usual equations of electrodynamics to charge carriers in a solid, we must use altered values of particle mass. In doing so, we account for most of the influences of the lattice, so that the electrons and holes can be treated as “almost free” carriers in most computations. The calculation of effective mass must take into account the shape of the energy bands in three-dimensional  $k$ -space, taking appropriate averages over the various energy bands.

**Example:** Find the  $(E, k)$  relationship for a free electron and relate it to the electron mass.

**Solution:**

The electron momentum is  $p = mv = \hbar k$ . Then

$$E = \frac{1}{2}mv^2 = \frac{1}{2} \frac{p^2}{m} = \frac{\hbar^2}{2m} k^2$$



Thus the electron energy is parabolic with wave vector  $k$ .

The electron mass is inversely related to the curvature (second derivative) of the  $(E, k)$

relationship, since 
$$\frac{d^2 E}{dk^2} = \frac{\hbar^2}{m}.$$

Although electrons in solids are not free, most energy bands are close to parabolic at their minima (for conduction bands) or maxima (for valence bands). We can also approximate effective mass near those band extrema from the curvature of the band.

The effective mass of an electron in a band with a given  $(E, k)$  relationship is given by

$$m^* = \frac{\hbar^2}{d^2 E / dk^2}$$

A particularly interesting feature is that the curvature  $d^2 E / dk^2$  is positive at the conduction band minima, and is negative at the valence band maxima. Thus, the electrons near the top of the valence band have **negative effective mass**. Valence band electrons with negative charge and negative mass move in an electric field in the same direction as holes with positive charge and positive mass. We can fully account for charge transport in the valence band by considering hole motion.

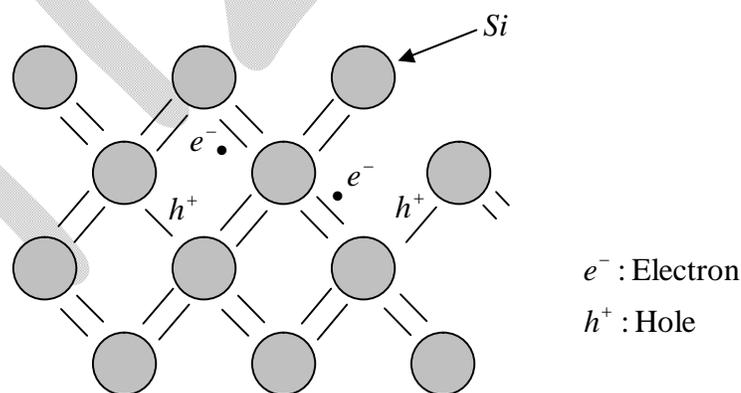
In any calculation involving the mass of the charge carriers, we must use effective mass values for the particular material involved. Table given below lists the effective masses for *Ge*, *Si*, and *GaAs* appropriate for one type of calculation. In this table and in all subsequent discussions, the electron effective mass is denoted by  $m_n^*$  and the hole effective mass by  $m_p^*$ . The *n* subscript indicates the electron as a negative charge carrier, and the *p* subscript indicates the hole as a positive charge carrier (The free electron rest mass is  $m_0$ ).

	<i>Ge</i>	<i>Si</i>	<i>GaAs</i>
$m_n^*$	$0.55 m_0$	$1.1 m_0$	$0.067 m_0$
$m_p^*$	$0.37 m_0$	$0.56 m_0$	$0.48 m_0$

### 1.4 Intrinsic Material

A perfect semiconductor crystal with no impurities or lattice defects is called an intrinsic semiconductor. In such material there are no charge carriers at  $0K$ , since the valence band is filled with electrons and the conduction band is empty. At higher temperatures electron-hole pairs are generated as valence band electrons are excited thermally across the band gap to the conduction band. These EHPs are the only charge carriers in intrinsic material.

The generation of EHPs can be visualized in a qualitative way by considering the breaking of covalent bonds in the crystal lattice. If one of the  $Si$  valence electrons is broken away from its position in the bonding structure such that it becomes free to move about in the lattice, a conduction electron is created and a broken bond (hole) is left behind. The energy required to break the bond is the band gap energy  $E_g$ . This model helps in visualizing the physical mechanism of EHP creation, but the energy band mode is more productive for purposes of quantitative calculation. One Important difficulty in the “broken bond” model is that the free electron and the hole seem deceptively localized in the lattice. Actually, the positions of the free electron and the hole are spread out over several lattice spacing and should be considered quantum mechanically by probability distributions.



**Figure 1.4:** Electron-hole pairs in the covalent bonding model of the  $Si$  crystal.

Since the electrons and holes are created in pairs, the conduction band electron concentration  $n$  (electrons per  $\text{cm}^3$ ) is equal to the concentration of holes in the valence band  $p$  (holes per  $\text{cm}^3$ ). Each of these intrinsic carrier concentrations is commonly referred to as  $n_i$ . Thus for intrinsic material

$$n = p = n_i.$$

At a given temperature there is a certain concentration of electron-hole pairs  $n_i$ . Obviously, if a steady state carrier concentration is maintained, there must be *recombination* of EHPs at the same rate at which they are generated. Recombination occurs when an electron in the conduction band makes a transition (direct or indirect) to an empty state (hole) in the valence band, thus annihilating the pair. If we denote the generation rate of EHPs as  $g_i$ , ( $\text{EHP}/\text{cm}^3$ ) and the recombination rate as  $r_i$ , equilibrium requires that:

$$r_i = g_i$$

Each of these rates is temperature dependent. For example,  $g_i(T)$  increases when the temperature is raised, and a new carrier concentration  $n_i$ , is established such that the higher recombination rate  $r_i(T)$  just balances generation. At any temperature, we can predict that the rate of recombination of electrons and holes  $r_i$  is proportional to the equilibrium concentration of electrons  $n_0$  and the concentration of holes  $p_0$

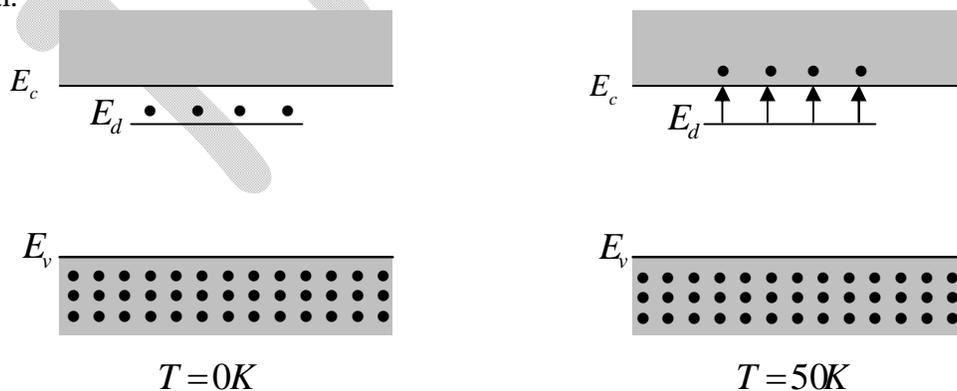
$$r_i = \alpha_r n_0 p_0 = \alpha_r n_i^2 = g_i$$

The factor  $\alpha_r$  is a constant of proportionality which depends on the particular mechanism by which recombination takes place.

### 1.5 Extrinsic Material

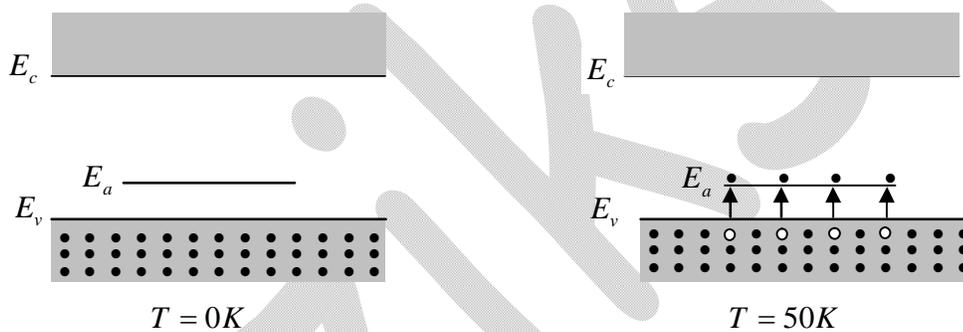
In addition to the intrinsic carriers generated thermally, it is possible to create carriers in semiconductor purposely by introducing impurities into the crystal. This process, called *doping* is the most common technique for varying the conductivity of semiconductors. By doping, a crystal can be altered so that it has a predominance of either electrons or holes. Thus there are two types of doped semiconductors, ***n*-type** (mostly electrons) and ***p*-type** (mostly holes).

When impurities or lattice defects are introduced into an otherwise perfect crystal, additional levels are created in the energy band structure usually within the band gap. For example, an impurity from column V of the periodic table (*P*, *As*, and *Sb*) introduces an energy level very near the conduction band in *Ge* or *Si*. This level is filled with electrons at  $0\text{ K}$ , and very little thermal energy is required to excite these electrons to the conduction band. Thus at about  $50\text{ K} - 100\text{ K}$  virtually all of the electrons in the impurity level are “donated” to the conduction band. Such an impurity level is called a **donor level** and the column V impurities in *Ge* or *Si* are called donor impurities. From figure 1.5, we note that the material doped with donor impurities can have a considerable concentration of electrons in the conduction band, even when the temperature is too low for the intrinsic EHP concentration to be appreciable. Thus semiconductors doped with a significant number of donor atoms will have  $n_0 \gg (n_i, p_0)$  at room temperature. This is ***n*-type** material.



**Figure 1.5:** Donation of electrons from a donor level to the conduction band.

Atoms from column III (*B*, *Al*, *Ga*, and *In*) introduce impurity levels in *Ge* or *Si* near the valence band. These levels are empty of electrons at  $0\text{ K}$ . At low temperatures, enough thermal energy is available to excite electrons from the valence band into the impurity level, leaving behind holes in the valence band, since this type of impurity level “accepts” electrons from the valence band, it is called an **acceptor level**, and the column III impurities are acceptor impurities in *Ge* and *Si*. Figure 1.6 indicates, doping with acceptor impurities can create a semiconductor with a hole concentration  $p_0$  much greater than the conduction band electron concentration  $n_0$  (this is ***p*-type** material).



**Figure 1.6:** Acceptance of valence band electrons by an acceptor level, and the resulting creation of holes.

### 1.6 The Fermi Level

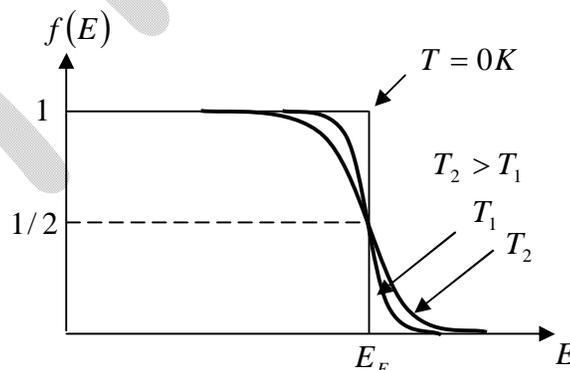
Electrons in solids obey *Fermi-Dirac* statistics. In the development of this type of statistics, one must consider the indistinguishability of the electrons, their wave nature, and the Pauli Exclusion Principle. The rather simple result of these statistical arguments is that the distribution of electrons over a range of allowed energy levels at thermal equilibrium is:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

where  $k$  is Boltzmann constant. The function  $f(E)$ , the *Fermi-Dirac distribution function*, gives the probability that an available energy state at  $E$  will be occupied by an electron at absolute temperature  $T$ . The quantity  $E_F$  is called the *Fermi Level*, and it represents an important quantity in the analysis of semiconductor behavior. We notice that, for an energy  $E$  equal to the Fermi level energy  $E_F$ , the occupation probability is

$$f(E_F) = \left[ 1 + e^{(E_F-E_F)/kT} \right]^{-1} = \frac{1}{1+1} = \frac{1}{2}.$$

A closer examination of  $f(E)$  indicates that at  $0K$  the distribution takes the simple rectangular form shown in figure 1.7. With  $T = 0$  in the denominator of the exponent,  $f(E)$  is  $1/(1+0) = 1$  when the exponent is negative ( $E < E_F$ ), and is  $1/(1+\infty) = 0$  when the exponent is positive ( $E > E_F$ ). This rectangular distribution implies that at  $0K$  every available energy state up to  $E_F$  is filled with electrons and all states above  $E_F$  are empty.



**Figure 1.7:** The Fermi Dirac distribution function.

At temperatures higher than  $0\text{ K}$ , some probability exists for states above  $E_F$  to be filled. For example, at  $T = T_1$  there is some probability  $f(E)$  that states above  $E_F$  are filled, and there is a corresponding probability  $[1 - f(E)]$  that states below  $E_F$  are empty.

The Fermi function is symmetrical about  $E_F$  for all temperatures; that is the probability  $f(E_F + \Delta E)$  that a state  $\Delta E$  above  $E_F$  is filled is the same as the probability  $[1 - f(E_F - \Delta E)]$  that a state  $\Delta E$  below  $E_F$  is empty. The symmetry of the distribution of empty and filled states about  $E_F$  makes the Fermi level a natural reference point in calculations of electron and hole concentrations in semiconductors.

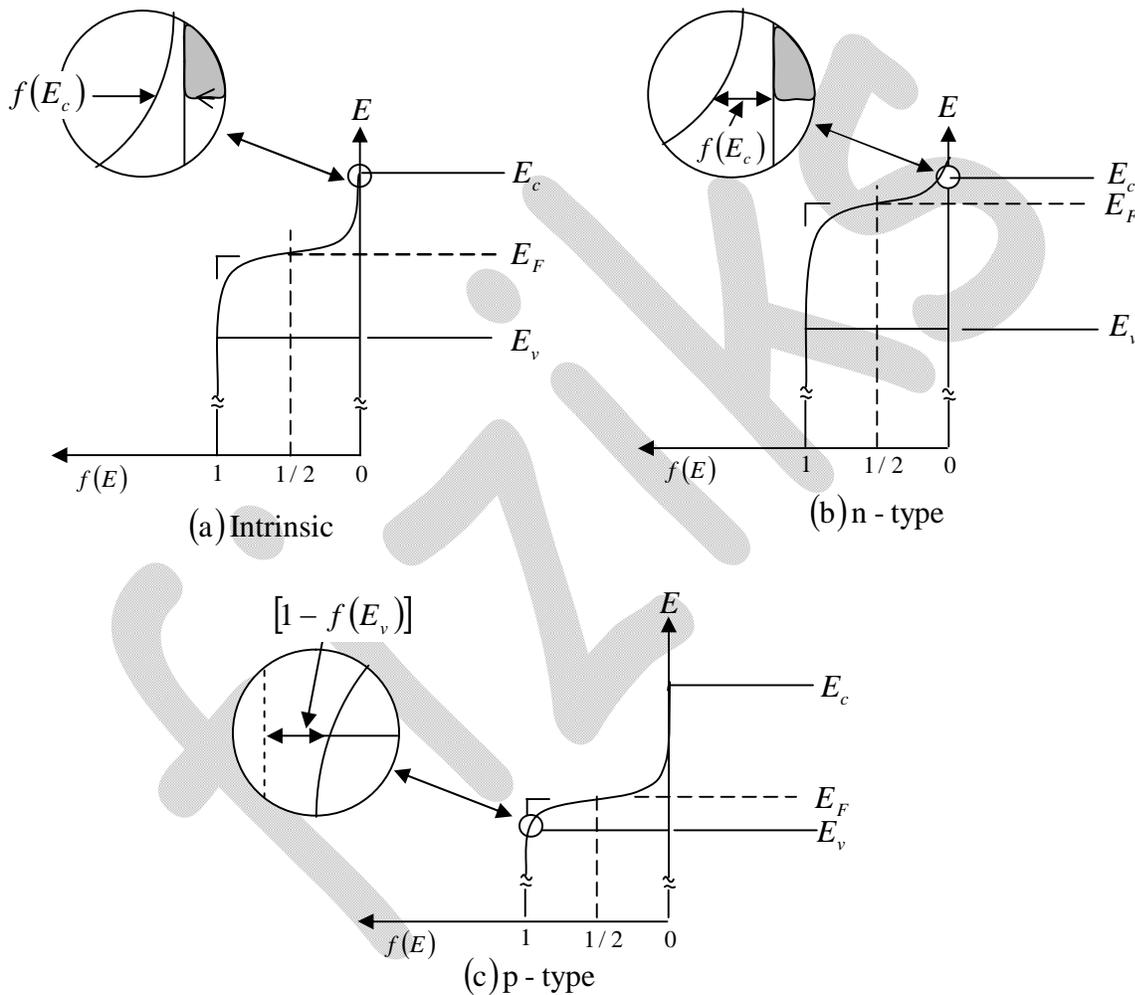
For intrinsic material we know that the concentration of holes in the valence band is equal to the concentration of electrons in the conduction band. Therefore, the Fermi level  $E_F$  must lie at the middle of the band gap in intrinsic material [Figure 1.8a]. Since  $f(E)$  is symmetrical about  $E_F$ , the electron probability "tail" of  $f(E)$  extending into the conduction band is symmetrical with the hole probability tail  $[1 - f(E)]$  in the valence band. The distribution function has values within the band gap between  $E_c$  and  $E_v$  but there are no energy states available, and no electron occupancy results from  $f(E)$  in this range.

In  $n$ -type material there is a high concentration of electrons in the conduction band compared with the hole concentration in the valence band. Thus in  $n$ -type material the distribution function  $f(E)$  must lie above its intrinsic position on the energy scale (figure 1.8 b). Since  $f(E)$  retains its shape for a particular temperature, the larger concentration of electrons at  $E_c$  in  $n$ -type material implies a correspondingly smaller hole concentration at  $E_v$ . We notice that the value of  $f(E)$  for each energy level in the conduction band (and therefore the total electron concentration  $n_0$ ) increases as  $E_F$  moves closer to  $E_c$ . Thus the energy difference  $(E_c - E_F)$  gives a measure of  $n$ .

For  $p$ -type material the Fermi level lies near the valence band (figure 1.8 c) such that the

$[1 - f(E)]$  tail below  $E_v$  is larger than the  $f(E)$  tail above  $E_c$ . The value of  $(E_F - E_v)$  indicates how strongly  $p$ -type the material is.

It is usually inconvenient to draw  $f(E)$  vs.  $E$  on every energy band diagram to indicate the electron and hole distributions. Therefore, it is common practice merely to indicate the position of  $E_F$  in band diagrams.



**Figure 1.8:** The Fermi distribution function applied to semiconductors:

(a) Intrinsic material; (b)  $n$ -type material; (c)  $p$ -type material.

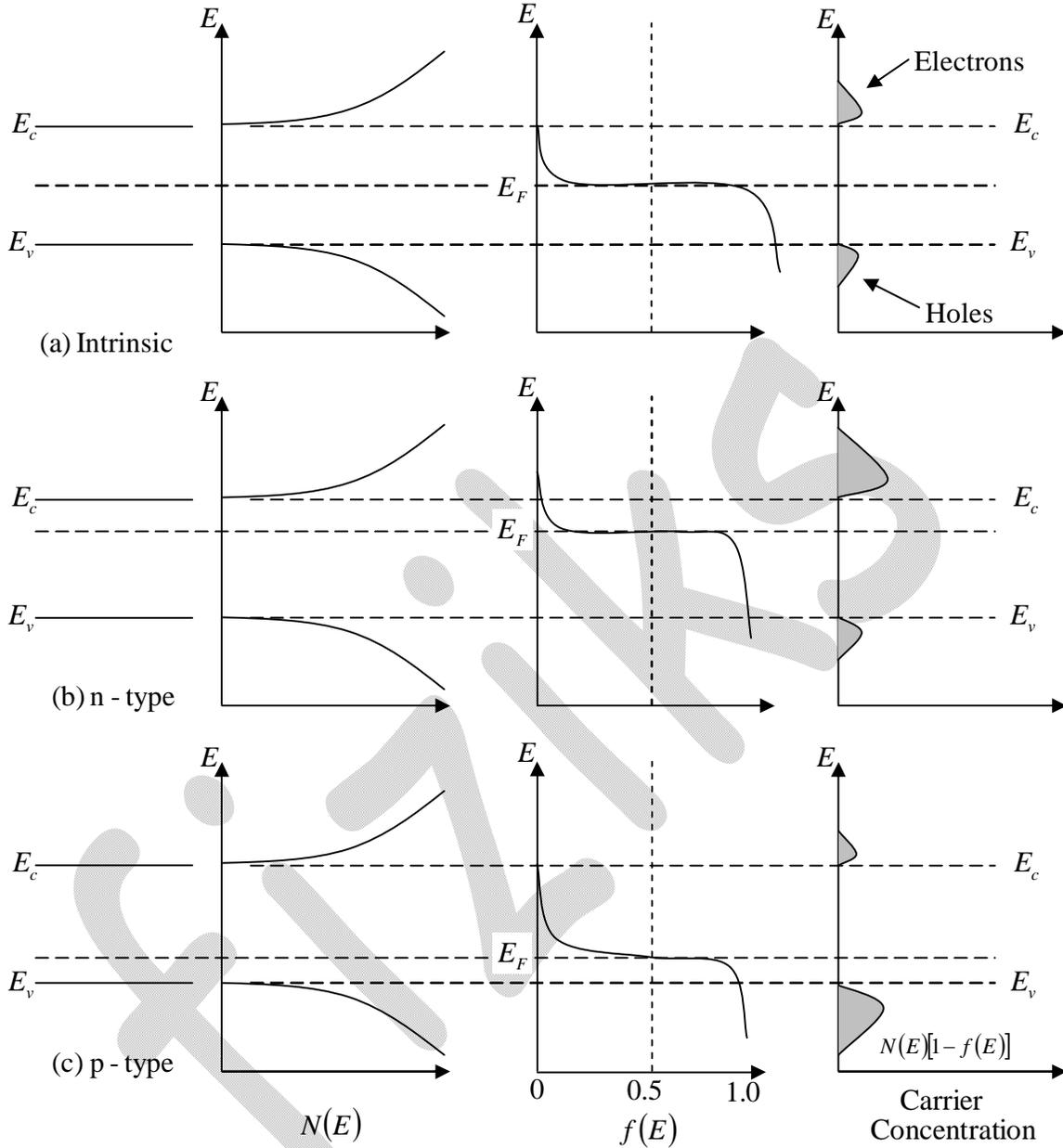
### 1.6.1 Electron and Hole Concentrations at Equilibrium

The Fermi distribution function can be used to calculate the concentrations of electrons and holes in a semiconductor, if the densities of available states in the valence and conduction bands are known. For example, the concentration of electrons in the conduction band is

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$$

where  $N(E)dE$  is the density of states ( $\text{cm}^{-3}$ ) in the energy range  $dE$ . The subscript 0 used with the electron and hole concentration symbols ( $n_0, p_0$ ) indicates equilibrium conditions. The number of electrons per unit volume in the energy range  $dE$  is the product of the density of states and the probability of occupancy  $f(E)$ . Thus the total electron concentration is the integral over the entire conduction band. The function  $N(E)$  can be calculated by using quantum mechanics and the Pauli Exclusion Principle.

Since  $N(E)$  is proportional to  $E^{1/2}$ , so the density of states in the conduction band increases with electron energy. On the other hand, the Fermi function becomes extremely small for large energies. The result is that the product  $f(E)N(E)$  decreases rapidly above  $E_c$  and very few electrons occupy energy states far above the conduction band edge. Similarly, the probability of finding an empty state (hole) in the valence band  $[1 - f(E)]$  decreases rapidly below  $E_v$  and most holes occupy states near the top of the valence band. This effect is demonstrated in figure 1.9, which shows the density of available states, the Fermi function, and the resulting number of electrons and holes occupying available energy states in the conduction and valence bands at thermal equilibrium (i.e., with no excitations except thermal energy). For holes, increasing energy points down, since the  $E$  scale refers to electron energy.



**Figure 1.9:** Schematic band diagram, density of states, Fermi-Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium.

The result of the integration of  $n_0 = \int_{E_c}^{\infty} f(E)N(E)dE$  is the same as that obtained if we represent the entire distributed electron states in the conduction band by an *effective density of states*  $N_c$  located at the conduction band edge  $E_c$ . Therefore, the conduction band electron concentration is simply the effective density of states at  $E_c$  times the probability of occupancy at  $E_c$

$$n_0 = f(E_c)N_c$$

In this expression we assume the Fermi level  $E_F$  lies at least several  $kT$  below the conduction band. Then the exponential term is large compared with unity and the Fermi function  $f(E_c)$  can be simplified as

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \approx e^{-(E_c - E_F)/kT}$$

Since  $kT$  at room temperature is only  $0.026 \text{ eV}$ , this is generally a good approximation. For this condition the concentration of electrons in the conduction band is

$$n_0 = N_c e^{-(E_c - E_F)/kT} \Rightarrow E_c - E_F = kT \ln \left( \frac{N_c}{n_0} \right)$$

The effective density of states  $N_c = 2 \left( \frac{2\pi m_n^* kT}{h^2} \right)^{3/2}$

Thus electron concentration increases as  $E_F$  moves closer to the conduction band.

By similar arguments, the concentration of holes in the valence band is

$$p_0 = N_v [1 - f(E_v)]$$

where  $N_v$  is the effective density of states in the valence band.

The probability of finding an empty state at  $E_v$  is,

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{-(E_v - E_F)/kT}} \approx e^{-(E_F - E_v)/kT} \text{ for } E_F \text{ larger than } E_v \text{ by several } kT.$$

From these equations, the concentration of holes in the valence band is

$$p_0 = N_v e^{-(E_F - E_v)/kT} \Rightarrow E_F - E_v = kT \ln \left( \frac{N_v}{p_0} \right)$$

The effective density of states in the valence band  $N_v = 2 \left( \frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$

Thus hole concentration increases as  $E_F$  moves closer to the valence band.

The electron and hole concentrations predicted by above equations are valid whether the material is intrinsic or doped, provided thermal equilibrium is maintained.

Thus for *intrinsic material*,  $E_F$  lies, at some intrinsic level  $E_i$  near the middle of the band gap, and the intrinsic electron and hole concentrations are

$$n_i = N_c e^{-(E_c - E_i)/kT}, \quad p_i = N_v e^{-(E_i - E_v)/kT}$$

$$n_i = p_i \Rightarrow E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \left( \frac{N_v}{N_c} \right) \Rightarrow E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln \left( \frac{m_p^*}{m_n^*} \right)$$

**Note:** The intrinsic level  $E_i$  is the middle of the band gap  $\left( E_c - E_i = \frac{E_g}{2} \right)$ , if the effective densities of states  $N_c$  and  $N_v$  are equal. There is usually some difference in effective mass for electrons and holes, however, and  $N_c$  and  $N_v$  are slightly different.

The product of  $n_0$  and  $p_0$  at equilibrium is a constant for a particular material and temperature, even if the doping is varied:

$$n_0 p_0 = \left( N_c e^{-(E_c - E_F)/kT} \right) \left( N_v e^{-(E_F - E_v)/kT} \right) = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$n_i p_i = \left( N_c e^{-(E_c - E_i)/kT} \right) \left( N_v e^{-(E_i - E_v)/kT} \right) = N_c N_v e^{-E_g/kT}$$

The intrinsic electron and hole concentrations are equal (since the carriers are created in pairs),  $n_i = p_i$ ; thus the intrinsic concentration is

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

## Law of Mass Action

The constant product of electron and hole concentrations can be written conveniently as

$$n_0 p_0 = n_i^2$$

For  $n$ -type material the minority concentration (holes)

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D} \quad \text{where } N_D \text{ is donor ion concentration.}$$

For  $p$ -type material the minority concentration (electrons)

$$n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A} \quad \text{where } N_A \text{ is acceptor ion concentration.}$$

Another convenient way of writing electron and hole concentration is

$$n_0 = n_i e^{(E_F - E_i)/kT} \quad \text{and} \quad p_0 = n_i e^{(E_i - E_F)/kT}$$

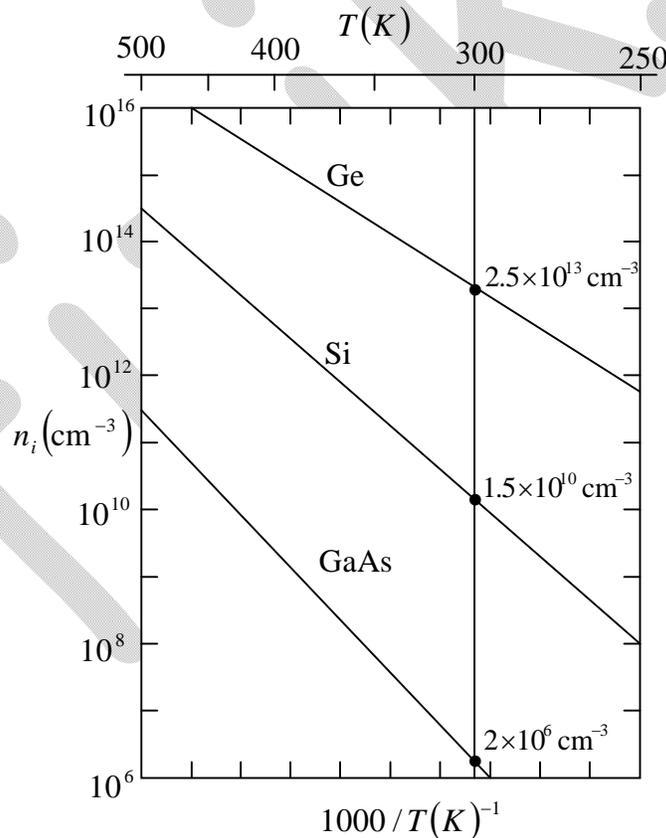
This form of the equation indicates directly that the electron concentration is  $n_i$  when  $E_F$  is at the intrinsic level  $E_i$  and that  $n_0$  increases exponentially as the Fermi level moves away from  $E_i$  toward the conduction band. Similarly, the hole concentration  $p_0$  varies from  $n_i$  to larger values as  $E_F$  moves from  $E_i$ , toward the valence band. Since these equations reveal the qualitative features of carrier concentration so directly, they are particularly convenient to remember.

## 1.7 Temperature Dependence of Carrier Concentrations

The variation of carrier concentration with temperature is indicated by equations  $n_0 = n_i e^{(E_F - E_i)/kT}$  and  $p_0 = n_i e^{(E_i - E_F)/kT}$ . Initially, the variation of  $n_0$  and  $p_0$  with  $T$  seems relatively straightforward in these relations. The problem is complicated, however, by the fact that  $n_i$  has strong temperature dependence ( $n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$ ) and that  $E_F$  can also vary with temperature. Let us begin by examining the intrinsic carrier concentration.

$$n_i(T) = 2 \left( \frac{2\pi kT}{h^2} \right)^{3/2} (m_n^* m_p^*)^{3/4} e^{-E_g/2kT}$$

The exponential temperature dependence dominates  $n_i(T)$  and a plot of  $\ln(n_i)$  vs  $1000/T$  appears almost linear (figure 1.10).

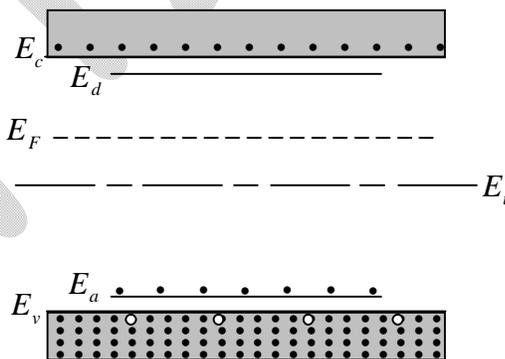


**Figure 1.10:** Intrinsic carrier concentration for *Ge*, *Si*, and *GaAs* as a function of inverse temperature. The room temperature values are marked for reference.

## 1.8 Compensation and Space Charge Neutrality

Figure 1.11 illustrates a semiconductor for which both donors and acceptors are present, but  $N_D > N_A$ . The predominance of donors makes the material  $n$ -type and the Fermi level is therefore in the upper part of the band gap. Since  $E_F$  is well above the acceptor level  $E_a$ , this level is essentially filled with electrons. However, with  $E_F$  above  $E_i$  we cannot expect a hole concentration in the valence band commensurate with the acceptor concentration. In fact, the filling of the  $E_a$  states occurs at the expense of the donated conduction band electrons.

The mechanism can be visualized as follows: Assume an acceptor state is filled with a valence band electron, with a hole resulting in the valence band. This hole is then filled by recombination with one of the conduction band electrons. Extending this logic to all the acceptor atoms, we expect the resultant concentration of electrons in the conduction band to be  $N_D - N_A$  instead of the total  $N_D$ . This process is called *compensation*. By this process it is possible to begin with an  $n$ -type semiconductor and add acceptors until  $N_A = N_D$  and no donated electrons remain in the conduction band. In such compensated material  $n_0 = n_i = p_0$  and intrinsic conduction is obtained. With further acceptor doping the semiconductor becomes  $p$ -type with a hole concentration of essentially  $N_A - N_D$ .



**Figure 1.11:** Compensation in an  $n$ -type semiconductor ( $N_D > N_A$ ).

The exact relationship among the electron, hole, donor, and acceptor concentrations can be obtained by considering the requirements for *space charge neutrality*. If the material is to remain electrostatically neutral, the sum of the positive charges (holes and ionized donor atoms) must balance the sum of the negative charges (electrons and ionized acceptor atoms):

$$p_0 + N_D^+ = n_0 + N_A^-$$

Thus the net electron concentration in the conduction band is  $n_0 = p_0 + (N_D^+ - N_A^-)$ .

If the material is doped *n*-type ( $n_0 \gg p_0$ ) and all the impurities are ionized, we can approximate that  $n_0 = N_D - N_A$ .

Since the intrinsic semiconductor itself is electrostatically neutral and the doping atoms we add are also neutral, the requirement of equation  $p_0 + N_D^+ = n_0 + N_A^-$  must be maintained at equilibrium.

Knowledge of carrier concentrations in a solid is necessary for calculating current flow in the presence of electric or magnetic fields. In addition to the values of  $n$  and  $p$ , we must be able to take into account the collisions of the charge carriers with the lattice and with the impurities. These processes will affect the ease with which electrons and holes can flow through the crystal, that is, their mobility within the solid. As should be expected, these collision and scattering processes depend on temperature, which affects the thermal motion of the lattice atoms and the velocity of the carriers.

**Example:** The donor concentration in a sample of  $n$ -type silicon is increased by a factor of 100. Find the shift in the position of the Fermi level at 300K. ( $k_B T = 25 \text{ meV}$  at 300K)

**Solution:**

$$E_C - E_F = kT \ln\left(\frac{N_c}{N_d}\right) \text{ and } E_C - E'_F = kT \ln\left(\frac{N_c}{100N_d}\right) = kT \ln\left(\frac{N_c}{N_d}\right) - kT \ln(100)$$

Thus shift is  $\Delta E = kT \ln(100) = 25 \ln(100) \text{ meV} = 115.15 \text{ meV}$

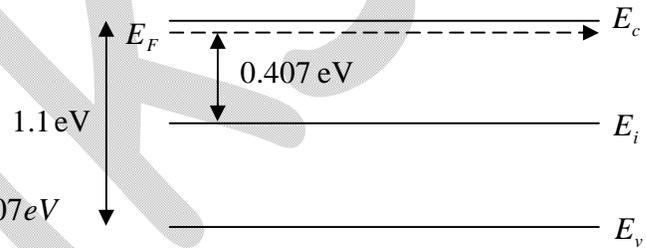
**Example:** A Si sample is doped with  $10^{17} \text{ As}$  atoms/ $\text{cm}^3$ . What is the equilibrium hole concentration  $p_0$  at 300 K? Where is  $E_F$  relative to  $E_i$ ? (where  $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ )

**Solution:**

Since  $N_D \gg n_i$  we can approximate  $n_i$  and

$$p_0 = \frac{n_i^2}{n_0} = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3}$$

$$E_F - E_i = kT \ln \frac{n_0}{n_i} = 0.0259 \ln \frac{10^{17}}{1.5 \times 10^{10}} = 0.407 \text{ eV}$$



**Example:** A pure Si sample at 300K with intrinsic carrier concentration of  $1.5 \times 10^{16} / \text{m}^3$  is doped with phosphorous. The equilibrium hole concentration and electron mobility is  $2.25 \times 10^9 / \text{m}^3$  and  $1350 \text{ cm}^2 / \text{Vs}$  respectively. Find the position of Fermi-level relative to the intrinsic level at 300K.

**Solution:**

Equilibrium electron concentration is

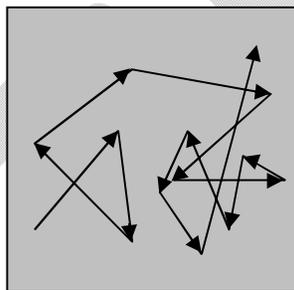
$$np = n_i^2 \quad (\text{Law of mass}) \Rightarrow n = \frac{n_i^2}{p} = \frac{(1.5 \times 10^{16})^2}{2.25 \times 10^9} = 1.00 \times 10^{23} \text{ m}^{-3}$$

$$E_F - E_i = k_B T \ln\left(\frac{n}{n_i}\right) = 8.67 \times 10^{-5} \times 300 \times \ln\left(\frac{10^{23}}{1.5 \times 10^{16}}\right) = 0.406 \text{ eV}$$

## 1.9 Current Components in Semiconductor

### 1.9.1 Drift Current (Conductivity and Mobility)

The charge carriers in a solid are in constant motion, even at thermal equilibrium. At room temperature, for example, the thermal motion of an individual electron may be visualized as random scattering from lattice atoms, impurities, other electrons, and defects (figure 1.12). Since the scattering is random, there is no net motion of the group of  $n$  electrons/cm<sup>3</sup> over any period of time. This is not true of an individual electron, of course. The probability of the electron in returning to its starting point after some time  $t$  is negligibly small. However, if a large number of electrons is considered (e.g.  $10^{16}$  cm<sup>-3</sup> in an  $n$ -type semiconductor), there will be no preferred direction of motion for the group of electrons and no net current flow.



**Figure 1.12:** Thermal motion of an electron in a solid.

If an electric field  $E_x$  is applied in the  $x$ -direction, each electron experiences a net force  $-qE_x$  from the field. This force may be insufficient to alter appreciably the random path of an individual electron; the effect when averaged over all the electrons, however, is a net motion of the group in the  $x$ -direction. If  $p_x$  is the  $x$ -component of the total momentum of the group, force of the field on the  $n$  electrons/cm<sup>3</sup> is

$$-nqE_x = \left. \frac{dp_x}{dt} \right|_{\text{field}} .$$

Initially, above equation seems to indicate a continuous acceleration of the electrons in the  $-x$ -direction. This is not the case, however, because the net acceleration is just balanced in steady state by the decelerations of the collision processes. Thus while the steady field  $E_x$  does produce a net momentum  $p_{-x}$ , the net rate of change of momentum when collisions are included must be zero in the case of steady state current flow.

To find the total rate of momentum change from collisions, we must investigate the collision probabilities more closely. If the collisions are truly random, there will be a constant probability of collision at any time for each electron. Let us consider a group of  $N_0$  electrons at time  $t=0$  and define  $N(t)$  as the number of electrons that have not undergone a collision by time  $t$ . The rate of decrease in  $N(t)$  at any time  $t$  is proportional to the number left unscattered at  $t$ ,

$$-\frac{dN(t)}{dt} = \frac{1}{\bar{t}} N(t) \quad \text{where } \bar{t}^{-1} \text{ is a constant proportionality.}$$

The solution to above equation is an exponential function  $N(t) = N_0 e^{-t/\bar{t}}$  and  $\bar{t}$  represents the mean time between scattering events, called the *mean free time*.

The probability that any electron has a collision in the time interval  $dt$  is  $\frac{dt}{\bar{t}}$ .

Thus the differential change in  $p$ , due to collisions in time  $dt$  is  $dp_x = -p_x \frac{dt}{\bar{t}}$ .

The rate of change of  $p_x$ , due to the decelerating effect of collisions is  $\left. \frac{dp_x}{dt} \right|_{\text{collision}} = -\frac{p_x}{\bar{t}}$ .

The sum of acceleration and deceleration effects must be zero for steady state. Thus

$$-\frac{p_x}{\bar{t}} - nqE_x = 0.$$

The average momentum per electron is  $\langle p_x \rangle = \frac{P_x}{n} = -q\bar{t} E_x$  where the angular brackets indicate an average over the entire group of electrons. As expected for steady state, the above equation indicates that the electrons have *on the average* a constant net velocity in

the negative  $x$ -direction:  $\langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = -\frac{q\bar{t}}{m_n^*} E_x$

Actually, the individual electrons move in many directions by thermal motion during a given time period, but  $\langle v_x \rangle$  tells us the *net drift* of an average electron in response to the electric field. The drift speed  $\langle v_x \rangle$  is usually much smaller than the random speed due to thermal motion  $v_{th}$ .

The **current density** resulting from this net drift is just the number of electrons crossing a unit area per unit time ( $n\langle v_x \rangle$ ) multiplied by the charge on the electron ( $-q$ ):

$$J_x = -qn\langle v_x \rangle = \frac{nq^2\bar{t}}{m_n^*} E_x \quad \text{ampere/cm}^2.$$

Thus the current density is proportional to the electric field, as we expect from Ohm's law:

$$J_x = \sigma E_x \quad \text{where } \sigma \equiv \frac{nq^2\bar{t}}{m_n^*}.$$

The **conductivity**  $\sigma$  ( $\Omega\text{-cm}$ )<sup>-1</sup> can be written  $\sigma = qn\mu_n$  where  $\mu_n \equiv \frac{q\bar{t}}{m_n^*}$ .

The quantity  $\mu_n$ , called the **electron mobility**, describes the ease with which electrons drift in the material. Mobility is a very important quantity in characterizing semiconductor materials and in device development.

The mobility can be expressed as the average particle drift velocity per unit electric field.

Thus  $\mu_n = -\frac{\langle v_x \rangle}{E_x}$ , and units of mobility are  $(\text{cm/s})/(\text{V/cm}) = \text{cm}^2/\text{V}\cdot\text{s}$ . The minus sign

in the definition results in a positive value of mobility, since electrons drift opposite to the field.

The current density can be written in terms of mobility as  $J_x = qn\mu_n E_x$ .

This derivation has been based on the assumption that the current is carried primarily by electrons. For hole conduction we change  $n$  to  $p$ ,  $-q$  to  $+q$ ,  $\mu_n$  to  $\mu_p$

where  $\mu_p = + \frac{\langle v_x \rangle}{E_x}$  is the mobility for holes.

If both electrons and holes participate, then

where  $\sigma = q(n\mu_n + p\mu_p)$ .

### For N-type semiconductor

$\sigma_n = e(n_n\mu_n + p_n\mu_p) \approx n_n e\mu_n$  since  $n_n \gg p_n$  where  $n_n$  and  $p_n$  are electron and hole concentration in N-type.

### For P-type semiconductor

$\sigma_p = e(n_p\mu_n + p_p\mu_p) \approx p_p e\mu_p$  since  $p_p \gg n_p$  where  $n_p$  and  $p_p$  are electron and hole concentration in P-type.

**Example:** The following data are given for intrinsic Germanium at 300 K .  
 $n_i = 2.4 \times 10^{19} / m^3$ ,  $\mu_e = 0.39 m^2 V^{-1} s^{-1}$ ,  $\mu_p = 0.19 m^2 V^{-1} s^{-1}$ . Find the conductivity of the Germanium.

**Solution:**  $\sigma = en_i(\mu_n + \mu_p) = 1.6 \times 10^{-19} \times 2.4 \times 10^{19} (0.39 + .19) = 2.227 (\Omega m)^{-1}$ .

**Example:** A sample of Si has electron and hole mobilities of 0.13 and 0.05  $m^2 V^{-1} s^{-1}$  respectively at 300K. It is doped with P and Al with doping densities of  $1.5 \times 10^{21} / m^3$  and  $2.5 \times 10^{21} / m^3$  respectively. The resistivity of doped Si sample at 300K is

- (a) 0.125  $\Omega m$       (b) 8.0  $\Omega m$       (c) 2.125  $\Omega m$       (d) 0.225  $\Omega m$

**Solution:**

Resulting doped crystal is p-type and  $p_p = (2.5 - 1.5) \times 10^{21} / m^3 = 1 \times 10^{21} / m^3$

$$\sigma = e(n_p\mu_n + p_p\mu_p) \approx ep_p\mu_p = 1.6 \times 10^{-19} \times 1 \times 10^{21} \times 0.05 = 8 \Omega^{-1} m^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{8} = 0.125 \Omega m$$

### 1.9.2 Diffusion Current

In addition to a conduction current, the transport of charges in a semiconductor may be accounted for a mechanism called diffusion. It is possible to have non-uniform concentration of particles in a semiconductor. As indicated in the figure 1.13, the concentration  $p$  of holes varies with distance  $x$  in the semiconductor, and there exist a

concentration gradient,  $\frac{dp}{dx}$  in the density of the carriers.

The existence of a gradient implies that if an imaginary surface is drawn in the semiconductor, the density of the holes immediately on one side of the surface is larger than the density on the other side. The holes are in random motion as a result of their thermal energy. Accordingly, holes will continue to move back and forth across this surface. We may then expect that, in a given time interval, more holes will cross the surface

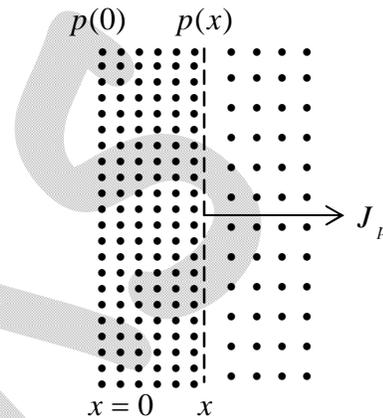


Figure 1.13

**Note:** It should be noted that this net transport of charge is not the result of mutual repulsion among charges of like sign, but is simply the result of a statistical phenomenon. This diffusion is exactly analogous to that which occurs in a neutral gas if concentration gradient exists in the gaseous container.

The diffusion hole-current density  $J_p$  (ampere per square meter) is proportional to the

concentration gradient, and is given by:  $J_p = -qD_p \frac{dp}{dx}$

where  $D_p$  (Square meters/second) is called diffusion constant. Since  $p$  decreases with

increasing  $x$ , then  $\frac{dp}{dx}$  is negative and the minus sign needed, so that  $J_p$  is positive in the

positive  $x$ -direction.

Similarly,  $J_n = qD_n \frac{dn}{dx}$

### 1.9.3 Einstein Relationship

Since both diffusion and mobility are statistical thermodynamic phenomena,  $D$  and  $\mu$  are not independent. The relationship between them is given by

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = V_T \quad \text{where } V_T \text{ is the 'Volt-equivalent of temperature'.$$

$$V_T = \frac{kT}{q} = \frac{T}{11,600} V$$

$k \rightarrow$  Boltzmann constant in electron volts per degree Kelvin

At room temperature  $T = 300^0 K$ ,  $V_T = 0.026 V \Rightarrow \mu = 39D$

### 1.9.4 Total Current in a Semiconductor

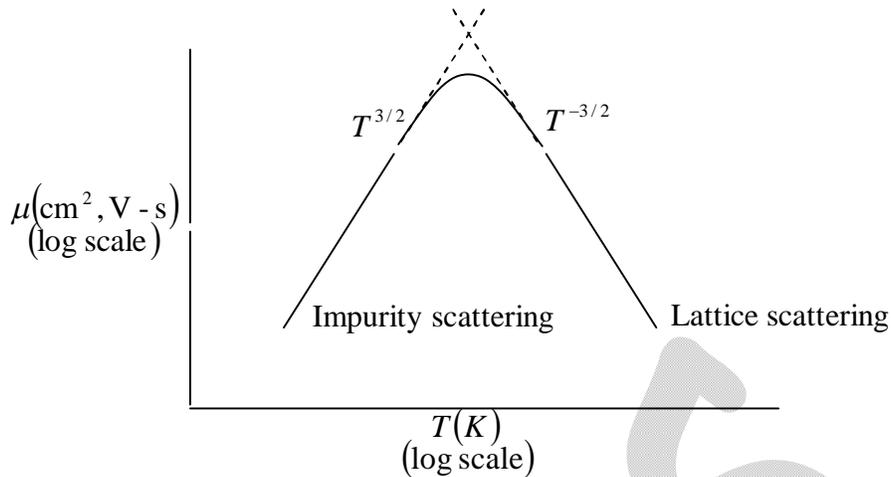
It is possible for both a potential gradient and a concentration gradient to exist simultaneously within a semiconductor. In such a situation, the total hole current is the

sum of the drift current and the diffusion current,  $J_p = q\mu_p pE - qD_p \frac{dp}{dx}$

Similarly the net electron current is:  $J_n = q\mu_n nE + qD_n \frac{dn}{dx}$

### 1.10 Effects of Temperature and Doping on Mobility

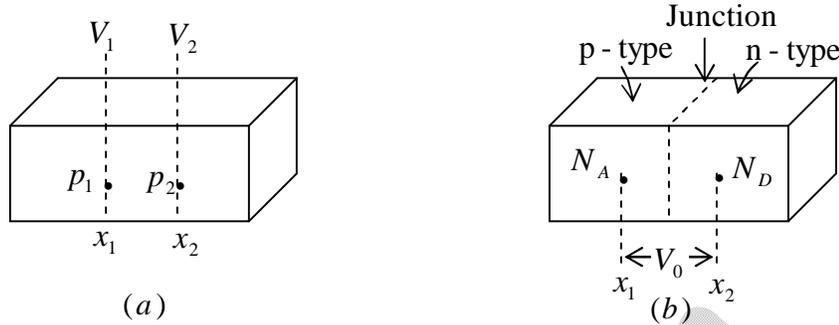
The two basic types of scattering mechanisms that influence electron and hole mobility are *lattice scattering* and *impurity scattering*. In lattice scattering a carrier moving through the crystal is scattered by a vibration of the lattice, resulting from the temperature (Collective vibrations of atoms in the crystal are called *phonons*. Thus lattice scattering is also known as *phonon scattering*). The frequency of such scattering events increases as the temperature increases, since the thermal agitation of the lattice becomes greater. Therefore, we should expect the mobility to decrease as the sample is heated. On the other hand, scattering from crystal defects such as ionized impurities becomes the dominant mechanism at low temperatures. Since the atoms of the cooler lattice are less agitated, lattice scattering is less important; however, the thermal motion of the carriers is also slower.



**Figure 1.14:** Approximate temperature dependence of mobility with both lattice and impurity scattering.

Since a slowly moving carrier is likely to be scattered more strongly by an interaction with a charged ion than is a carrier with greater momentum, impurity scattering events cause a decrease in mobility with decreasing temperature. The approximate temperature dependencies are  $T^{-3/2}$  for lattice scattering and  $T^{3/2}$  for impurity scattering.

## 1. 11 The Potential Variation within a Graded Semiconductor



**Figure 1.15** (a): A graded semiconductor:  $p(x)$  is not constant

(b): One portion is doped with (uniformly) acceptor ions and the other section is doped uniformly with donor ions so that a metallurgical junction is formed.

Consider a semiconductor where the hole concentration  $p$  is a function of  $x$ ; that is, the doping is non-uniform or graded. Assume a steady-state situation and zero excitation; that is, no carriers are injected into the specimen from any external source. With no excitation there can be no steady movement of charge in the bar, although the carriers possess random motion due to thermal agitation. Hence the total hole current must be zero (also, the total electron current must be zero). Since  $p$  is not constant, we expect a non-zero hole diffusion current. In order for the total hole current to vanish there must exist a hole drift current which is equal and opposite to the diffusion current. However, conduction current requires an electric field and hence we conclude that, as a result of the non-uniform doping, an electric field is generated within the semiconductor. We shall now find this field and the corresponding potential variation throughout the bar.

$$\text{Since } J_p = q\mu_p pE - qD_p \frac{dp}{dx} \Rightarrow E = \frac{V_T}{p} \frac{dp}{dx} \quad \because J_p = 0 \text{ and then use } D_p = \mu_p V_T$$

If the doping concentration  $p(x)$  is known, this equation allows the built in field  $E(x)$  to be calculated.

$$\because E = -\frac{dV}{dx} \Rightarrow dV = -V_T \frac{dp}{p}.$$

If this equation is integrated between  $x_1$ , where the concentration is  $p_1$  and the potential

is  $V_1$  and  $x_2$  where  $p = p_2$  and  $V = V_2$ , the result is:  $V_{21} \equiv V_2 - V_1 = V_T \ln \frac{p_1}{p_2}$

**Note:** The potential difference between two points depends only upon the concentration at these points and is independent of their separation ( $x_2 - x_1$ ).

Above equation can be put in the form  $p_1 = p_2 e^{V_{21}/V_T}$

This is the Boltzmann relationship of kinetic gas theory.

Starting with  $J_n = 0$  and proceeding as above, the Boltzmann equation for electrons is obtained as  $n_1 = n_2 e^{-V_{21}/V_T}$ . Now  $n_1 p_1 = n_2 p_2$ .

This equation states that the product  $np$  is a constant independent of  $x$ , and hence the amount of doping, under thermal equilibrium.

For an intrinsic semiconductor  $n = p = n_i$  and hence  $np = n_i^2$ .

### 1.11.1 An Open-Circuited Step-graded Junction

Consider the special case indicated in figure 1.15 (b). The left half of the bar is  $p$ -type with a constant concentration  $N_A$ , whereas the right-half is  $n$ -type with a uniform density  $N_D$ . The dashed plane is a metallurgical ( $p-n$ ) junction separating the two sections with different concentrations. This type of doping where the density changes abruptly from  $p$  to  $n$  type is called step-grading. The step graded junction is located at the plane where the concentration is zero. The above theory indicates that there is built-in potential between these two sections (called the contact difference of potential  $V_o$ .)

$$\text{Thus } V_o = V_{21} = V_T \ln \frac{p_{p_o}}{p_{n_o}}$$

Because  $p_1 = p_{p_o}$  = thermal-equilibrium hole concentration in  $p$ -side

$p_2 = p_{n_o}$  = thermal equilibrium hole concentration in  $n$ -side

$$\text{since } p_{p_o} = N_A \text{ and } p_{n_o} = \frac{n_i^2}{N_D} \Rightarrow \boxed{V_o = V_T \ln \frac{N_A N_D}{n_i^2}}$$

## Summary

1. In a semiconductor two types of mobile charge carriers are available. The bipolar nature of a semiconductor is to be contrasted with the unipolar property of a metal, which possesses only free electrons.
2. A semiconductor may be fabricated with donor (acceptor) impurities. So it contains mobile charges which are primarily electrons (holes).
3. The intrinsic carrier concentration is a function of temperature. At room temperature, essentially all donors or acceptors are ionized.
4. Current is due to two distinct phenomena:
  - (a) Carriers drift in an electric field (this conduction current is also available in metals).
  - (b) Carriers diffuse if a concentration gradient exists (a phenomenon, which does not take place in metals).
5. Carriers are continuously being generated (due to thermal creation of hole-electron pairs) and are simultaneously disappearing (due to recombination).
6. The fundamental law governing the flow of charges is called the *continuity equation*. It is formulated by considering that charges can neither be created nor destroyed if generation, recombination, drift and diffusion are all taken into account.
7. If the minority carriers are injected into a region containing majority carriers, then usually the injected minority concentration is very small compared with the density of the majority carriers. For this low-level injection condition the *minority current is predominantly due to diffusion*; in other words, the minority drift current may be neglected.
8. The total majority-carrier flow is the sum of a drift and diffusion current. The majority conduction current results from a small electric field internally created within the semiconductor because of the injected carriers.
9. The minority-carrier concentration injected into one end of a semiconductor bar decreases exponentially with distance into the specimen (as a result of diffusion and recombination).
10. Across an open-circuited p-n junction there exists a contact difference of potential.

Multiple Choice Questions (MCQ)

**Q1.** Consider the following statements: Electrical conductivity of a metal has negative temperature coefficient since

1. Electron concentration increases with temperature
2. Electron mobility decreases with temperature
3. Electron lattice scattering increases with temperature.

Which of the following statements given above correct?

- (a) 1, 2, 3                      (b) only 1 and 2                      (c) only 2 and 3                      (d) only 1 and 3

**Q2.** A piece of copper and a piece of germanium are cooled from room temperature to  $100\text{ K}$ . Then which one of the following is correct?

- (a) Resistance of each will increase
- (b) Resistance of each will decrease
- (c) Resistance of copper will increase while that of germanium will decrease.
- (d) Resistance of copper will decrease while that of germanium will increase.

**Q3.** The probability of electrons to be found in the conduction band of an intrinsic semiconductor at a finite temperature

- (a) Increases exponentially with increasing band gap.
- (b) Decreases exponentially with increasing band gap.
- (c) Decreases with increasing temperature.
- (d) is independent of the temperature and the band gap.

**Q4.** Pure silicon at  $300\text{ K}$  has equal electron and hole concentration of  $2 \times 10^{16}\text{ m}^{-3}$ . It is doped by indium to the extent one part in  $10^7$  silicon atom. If the density of silicon is  $4 \times 10^{29}\text{ m}^{-3}$ , then the electron concentration in the doped silicon is

- (a)  $10^5\text{ m}^{-3}$                       (b)  $10^7\text{ m}^{-3}$                       (c)  $10^9\text{ m}^{-3}$                       (d)  $10^{10}\text{ m}^{-3}$

**Q5.** Two pure specimen of a semiconductor material are taken. One is doped with  $10^{18} \text{ cm}^{-3}$  numbers of donors and the other is doped with  $10^{16} \text{ cm}^{-3}$  numbers of acceptors. The minority carrier density in the first specimen is  $10^7 \text{ cm}^{-3}$ . What is the minority carrier density in the other specimen?

- (a)  $10^{16} \text{ cm}^{-3}$       (b)  $10^{27} \text{ cm}^{-3}$       (c)  $10^{18} \text{ cm}^{-3}$       (d)  $10^9 \text{ cm}^{-3}$

**Q6.** The donor concentration in a sample of  $n$ -type silicon is increased by a factor of 100. The shift in the position of the Fermi level at  $300 \text{ K}$ , assuming the sample to non degenerate is ( $k_B T = 25 \text{ meV}$  at  $300 \text{ K}$ )

- (a)  $105 \text{ meV}$       (b)  $110 \text{ meV}$       (c)  $115 \text{ meV}$       (d)  $120 \text{ meV}$

**Q7.** A sample of  $Si$  has electron and hole mobility's of  $0.13$  and  $0.05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  respectively at  $300 \text{ K}$ . It is doped with  $P$  and  $Al$  with doping densities of  $1.5 \times 10^{21} / \text{m}^3$  and  $2.5 \times 10^{21} / \text{m}^3$  respectively. The conductivity of doped  $Si$  sample at  $300 \text{ K}$  is

- (a)  $8 \Omega^{-1} \text{ m}^{-1}$       (b)  $32 \Omega^{-1} \text{ m}^{-1}$       (c)  $20.8 \Omega^{-1} \text{ m}^{-1}$       (d)  $83.2 \Omega^{-1} \text{ m}^{-1}$

**Q8.** A sample of  $Si$  has electron and hole mobility's of  $0.13$  and  $0.05 \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}$  respectively at  $300 \text{ K}$ . It is doped with  $P$  and  $Al$  with doping densities of  $2.5 \times 10^{21} / \text{m}^3$  and  $1.5 \times 10^{21} / \text{m}^3$  respectively. The conductivity of doped  $Si$  sample at  $300 \text{ K}$  is

- (a)  $8 \Omega^{-1} \text{ m}^{-1}$       (b)  $32 \Omega^{-1} \text{ m}^{-1}$       (c)  $20.8 \Omega^{-1} \text{ m}^{-1}$       (d)  $83.2 \Omega^{-1} \text{ m}^{-1}$

**Q9.** Mobility of electrons as well as holes for intrinsic germanium is given by  $3900 \text{ cm}^2 / \text{V} - \text{sec}$  and  $1900 \text{ cm}^2 / \text{V} - \text{sec}$  with intrinsic concentration  $2.5 \times 10^{13} \text{ cm}^{-3}$ . Then the intrinsic resistivity of the material is

- (a)  $43 \Omega \text{ cm}$       (b)  $64 \Omega \text{ cm}$       (c)  $86 \Omega \text{ cm}$       (d)  $131 \Omega \text{ cm}$

**Q10.** Consider an extrinsic semiconductor with intrinsic concentration of  $n_i$ . If  $\mu_p$  and  $\mu_n$  are mobility of holes and electron then the electron concentration at which semiconductor have minimum conductivity and  $\sigma_{\min}$  are

- (a)  $n_i\sqrt{\mu_p\mu_n}$ ,  $n_ie\sqrt{\mu_p\mu_n}$                       (b)  $n_i\sqrt{\mu_p/\mu_n}$ ,  $2n_ie\sqrt{\mu_p\mu_n}$   
 (c)  $n_i\sqrt{\mu_n/\mu_p}$ ,  $n_ie\sqrt{\mu_n/\mu_p}$                       (d)  $n_i/\sqrt{\mu_n\mu_p}$ ,  $2n_ie/\sqrt{\mu_n\mu_p}$

**Q11.** The Fermi-level in an  $n$ -type and  $p$ -type semiconductor material is expressed as (where  $N_D$ ,  $N_A$  are donor and acceptor ion concentration and  $N_C$ ,  $N_V$  are effective density of states in conduction and valance band)

- (a)  $E_C + kT \ln\left(\frac{N_D}{N_C}\right)$ ,  $E_V + kT \ln\left(\frac{N_A}{N_V}\right)$                       (b)  $E_C - kT \ln\left(\frac{N_D}{N_C}\right)$ ,  $E_V - kT \ln\left(\frac{N_A}{N_V}\right)$   
 (c)  $E_C + kT \ln\left(\frac{N_D}{N_C}\right)$ ,  $E_V - kT \ln\left(\frac{N_A}{N_V}\right)$                       (d)  $E_C - kT \ln\left(\frac{N_D}{N_C}\right)$ ,  $E_V + kT \ln\left(\frac{N_A}{N_V}\right)$

**Q12.** In an  $n$ -type semiconductor the minority hole concentration is  $p_n$  and intrinsic carrier concentration is  $n_i$ . If the effective density of state in conduction band is  $n_c$  at temperature  $T^0K$ . Then relative position of the Fermi-level with respect to level ( $E_C$ ) is

- (a)  $KT \ln \frac{n_c}{n_i^2 p_n}$                       (b)  $KT \ln \frac{n_i^2 \cdot n_c}{p_n}$   
 (c)  $KT \ln \frac{p_n \cdot n_c}{n_i^2}$                       (d)  $KT \ln \frac{n_i^2}{p_n \cdot n_c}$

**Q13.** A  $Si$  sample ( $n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$ ) is doped with  $10^{17} \text{ As atoms / cm}^3$ . Then

relative position of Fermi-level ( $E_F$ ) with respect to intrinsic level ( $E_i$ ) is

- (a) 0.12 eV                      (b) 0.14 eV                      (c) 0.16 eV                      (d) 0.41 eV

**Q14.** In an  $n$ -type semiconductor, the Fermi level is  $0.24 \text{ eV}$  below the conduction band at room temperature of  $300 \text{ K}$ . If the temperature is increased to  $350 \text{ K}$ , then the new position of the Fermi-level is (Assume effective density of states to be independent of temperature):

- (a)  $0.28 \text{ eV}$                       (b)  $0.38 \text{ eV}$                       (c)  $0.48 \text{ eV}$                       (d)  $0.58 \text{ eV}$

**Q15.** A  $p$ -type semiconductor (acceptor ion concentration is  $N_A$ ) is doped with donor ion (concentration is  $N_D$ ) and  $N_D > N_A$ . If the intrinsic concentration is  $n_i$ , then the concentration of minority carrier in the doped specimen will be:

- (a)  $\frac{n_i^2}{N_D}$                       (b)  $\frac{n_i^2}{N_A}$                       (c)  $\frac{n_i^2}{(N_D - N_A)}$                       (d)  $\frac{n_i^2}{(N_A - N_D)}$

**Numerical Answer Type Questions (NAT)**

**Q16.** Pure silicon at  $300 \text{ K}$  has equal electron and hole concentration of  $1.5 \times 10^{16} \text{ m}^{-3}$ . Doping by indium increases hole concentration to  $4.5 \times 10^{22} \text{ m}^{-3}$ . Then the electron concentration in the doped silicon is..... $\times 10^9 \text{ m}^{-3}$

- (a)  $9 \times 10^5 \text{ m}^{-3}$                       (b)  $5 \times 10^9 \text{ m}^{-3}$                       (c)  $9 \times 10^{-5} \text{ m}^{-3}$                       (d)  $5 \times 10^{-9} \text{ m}^{-3}$

**Q17.** A pure  $\text{Si}$  sample at  $300 \text{ K}$  with intrinsic carrier concentration of  $1.5 \times 10^{16} / \text{m}^3$  is doped with phosphorous. The equilibrium hole concentration and electron mobility is  $2.25 \times 10^9 / \text{m}^3$  and  $1350 \text{ cm}^2 / \text{Vs}$  respectively. Then the Position of Fermi-level relative to the intrinsic level at  $300 \text{ K}$  is..... $\text{eV}$

**Q18.** A sample of  $\text{Si}$  has electron and hole mobility's of  $0.13$  and  $0.05 \text{ m}^2 \text{V}^{-1} \text{ s}^{-1}$  respectively at  $300 \text{ K}$ . It is doped with  $\text{P}$  and  $\text{Al}$  with doping densities of  $2.5 \times 10^{21} / \text{m}^3$  and  $1.5 \times 10^{21} / \text{m}^3$  respectively. The resistivity of doped  $\text{Si}$  sample at  $300 \text{ K}$  is..... $\Omega \text{m}$

**Q19.** The following data are given for intrinsic Germanium at  $300\text{ K}$  .  $n_i = 2.4 \times 10^{19} / \text{m}^3$  ,  $\mu_e = 0.39 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$  ,  $\mu_p = 0.19 \text{m}^2 \text{V}^{-1} \text{s}^{-1}$  . The resistivity of the Germanium will turn out to be ..... $\Omega \text{m}$

**Q20.** A semiconductor has following parameters  $\mu_n = 7500 \text{ cm}^2 / \text{Vs}$  ,  $\mu_p = 300 \text{ cm}^2 / \text{Vs}$  and  $n_i = 3.6 \times 10^{12} \text{ cm}^{-3}$  . When the conductivity is minimum, the hole concentration is ..... $\times 10^{13} \text{ cm}^{-3}$

### Multiple Select Type Questions (MSQ)

**Q21.** Which of the following are true regarding Fermi-Dirac distribution function

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

- (a) An energy state at the Fermi level has a probability of  $1/2$  of being occupied by an electron.
- (b) At  $0\text{ K}$  , every available energy state up to  $E_F$  is filled with electrons, and all states above  $E_F$  are empty.
- (c) At temperatures higher than  $0\text{ K}$  , there is some probability  $f(E)$  that states above  $E_F$  are filled and there is a corresponding probability  $[1 - f(E)]$  that states below  $E_F$  are empty.
- (d) The Fermi function is unsymmetrical about  $E_F$  for all temperatures.

**Q22.** Which of the following statement are true regarding semiconductors?

- (a) An  $n$ -type semiconductor behaves as an intrinsic semiconductor at very high temperature.
- (b) The breaking of the covalent bonds becomes a significant phenomenon at high temperatures.
- (c) The carriers mobility increases with increase of temperature.
- (d) The carriers mobility decreases with increase of temperature.

**Q23.** Which one of the following are true?

- (a) Metals have positive temperature coefficient of resistance.
- (b) Semiconductors have negative temperature coefficient of resistance.
- (c) Conductivity of metals decreases with increase in temperature.
- (d) Conductivity of semiconductor decreases with increase in temperature.

**Q24.** Which one of the following are not true?

- (a) The diffusion constants  $D_n$  and  $D_p$  for electrons and holes respectively are related

to their mobility by Einstein equation  $\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{e}$

- (b) The expression  $n \cdot p = n_i^2$  is valid for semiconductors at all temperature.
- (c)  $n_i(T) = AT^{3/2} \exp\left(-\frac{E_g}{2kT}\right)$ , correctly describe the temperature ( $T$ ) variation of the intrinsic carrier density of a semiconductor
- (d) Gallium Arsenide ( $GaAs$ ) is an indirect band gap semiconductor with  $E_g = 1.43eV$  at room temperature

**Q25.** Which of the following are true?

- (a)  $Si$  and  $Ge$  are indirect band gap semiconductor.
- (b) At  $300^0 K$  the band gap energies of  $Si$  and  $Ge$  are  $1.1eV$  and  $0.72eV$ .
- (b) At  $0^0 K$  the band gap energies of  $Si$  and  $Ge$  are  $1.1eV$  and  $0.72eV$ .
- (d) At  $300^0 K$  mobility ( $\mu$ ) and diffusion constant ( $D$ ) is related by  $\mu = 39D$ .

## Solution

### MCQ

**Ans.1:** (c)

**Ans.2:** (d)

**Ans.3:** (b)

**Ans.4:** (d)

$$\text{Acceptor ion concentration } N_A \approx p = \frac{4 \times 10^{29} \text{ m}^{-3}}{10^7} = 4 \times 10^{22} \text{ m}^{-3}$$

According to Law of Mass Action,  $n \cdot p = n_i^2$

$$n \cdot p = n' \cdot p' \Rightarrow n' = \frac{n \cdot p}{p'} = \frac{(2 \times 10^{16}) \times (2 \times 10^{16})}{(4 \times 10^{22})} = 10^{10} \text{ m}^{-3}$$

**Ans.5:** (d)

According to law of mass action,

$$n_1 \cdot p_1 = n_2 \cdot p_2 \Rightarrow n_2 = \frac{n_1 \cdot p_1}{p_2} = \frac{10^{18} \times 10^7}{10^{16}} = 10^9 \text{ cm}^{-3}$$

**Ans.6:** (c)

$$E_C - E_F = kT \ln \left( \frac{N_c}{N_d} \right) \text{ and } E_C - E'_F = kT \ln \left( \frac{N_c}{100N_d} \right) = kT \ln \left( \frac{N_c}{N_d} \right) - kT \ln(100)$$

Thus shift is  $\Delta E = kT \ln(100) = 25 \ln(100) \text{ meV} = 115.15 \text{ meV}$

**Ans.7:** (a)

Resulting doped crystal is  $p$ -type and  $p_p = (2.5 - 1.5) \times 10^{21} / \text{m}^3 = 1 \times 10^{21} / \text{m}^3$

$$\sigma = e(n_p \mu_n + p_p \mu_p) \approx e p_p \mu_p = 1.6 \times 10^{-19} \times 1 \times 10^{21} \times 0.05 = 8 \Omega^{-1} \text{ m}^{-1}$$

**Ans.8:** (c)

Resulting doped crystal is  $n$ -type and  $n_n = (2.5 - 1.5) \times 10^{21} / \text{m}^3 = 1 \times 10^{21} / \text{m}^3$

$$\sigma = e(n_n \mu_n + p_n \mu_p) \approx e n_n \mu_n = 1.6 \times 10^{-19} \times 1 \times 10^{21} \times 0.13 = 20.8 \Omega^{-1} \text{ m}^{-1}$$

**Ans.9: (a)**

$$\rho_i = \frac{1}{\sigma_i} = \frac{1}{n_i e (\mu_e + \mu_h)} = \frac{1}{2.5 \times 10^{13} \times 1.6 \times 10^{-19} \times (5800)} = 43 \Omega \text{cm}$$

**Ans.10: (b)**

$$\text{Conductivity } \sigma = e(n\mu_n + p\mu_p) = e \left( n\mu_n + \frac{n_i^2}{n} \mu_p \right)$$

$$\text{For minimum conductivity, } \frac{d\sigma}{dn} = 0 \Rightarrow n = n_i \sqrt{\mu_p / \mu_n}$$

$$\text{Thus } \sigma_{\min} = 2n_i e \sqrt{\mu_p \mu_n}$$

**Ans.11: (c)**

**Ans.12: (c)**

$$n_n = N_c e^{-(E_c - E_F)/kT} \Rightarrow E_c - E_F = kT \ln \left( \frac{N_c}{n_n} \right) \text{ and } n_n \cdot p_n = n_i^2$$

**Ans.13: (d)**

$$p_n = \frac{n_i^2}{n_n} = \frac{2.25 \times 10^{20}}{10^{17}} = 2.25 \times 10^3 \text{ cm}^{-3} \Rightarrow E_F - E_i = kT \ln \frac{n_n}{n_i} = 0.407 \text{ eV}$$

**Ans.14: (a)**

$$E_C - E_F = kT \ln \frac{N_C}{N_D} \Rightarrow 0.24 = 300k \ln \frac{N_C}{N_D} \text{ and } E_C - E'_F = 350k \ln \frac{N_C}{N_D}$$

$$\Rightarrow E_C - E'_F = 0.28 \text{ eV}$$

**Ans.15: (c)**

## NAT

**Ans.16: 5**

According to Law of Mass Action,  $n \cdot p = n_i^2$

$$n \cdot p = n' \cdot p' \Rightarrow n' = \frac{n \cdot p}{p'} = \frac{(1.5 \times 10^{16}) \times (1.5 \times 10^{16})}{(4.5 \times 10^{22})} = 5 \times 10^9 \text{ m}^{-3}$$

**Ans.17: 0.41**

Equilibrium electron concentration is

$$np = n_i^2 \quad (\text{Law of mass}) \Rightarrow n = \frac{n_i^2}{p} = \frac{(1.5 \times 10^{16})^2}{2.25 \times 10^9} = 1.00 \times 10^{23} \text{ m}^{-3}$$

$$E_F - E_i = k_B T \ln\left(\frac{n}{n_i}\right) = 8.67 \times 10^{-5} \times 300 \times \ln\left(\frac{10^{23}}{1.5 \times 10^{16}}\right) = 0.406 \text{ eV}$$

**Ans.18: 0.05**

Resulting doped crystal is  $n$ -type and  $n_n = (2.5 - 1.5) \times 10^{21} / \text{m}^3 = 1 \times 10^{21} / \text{m}^3$

$$\sigma = e(n_n \mu_n + p_n \mu_p) \approx en_n \mu_n = 1.6 \times 10^{-19} \times 1 \times 10^{21} \times 0.13 = 20.8 \Omega^{-1} \text{ m}^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{20.8} = 0.048 \Omega \text{ m}$$

**Ans.19: 0.45**

$$\sigma = en_i(\mu_n + \mu_p) = 1.6 \times 10^{-19} \times 2.4 \times 10^{19} (0.39 + .19) = 2.227 (\Omega \text{ m})^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{2.227} = 0.449 \Omega \text{ m}$$

**Ans.20: 2**

$$p = n_i \sqrt{\mu_n / \mu_p} = 3.6 \times 10^{12} \sqrt{7500 / 300} = 18 \times 10^{12} = 2 \times 10^{13} \text{ cm}^{-3}$$

## MSQ

**Ans.21: (a), (b) and (c)**

**Ans.22: (a), (b) and (d)**

**Ans.23: (a), (b) and (c)**

**Ans.24: (a), (b) and (c)**

**Ans.25: (a), (c) and (d)**