Solution

Gate Full Length Test - 01

15-01-2018

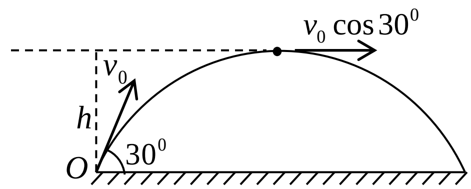
Ans. 1: (b)

Solution: $L = |J_i - J_f| ..... |J_i + J_f| = 1, 2, 3, 4$

For $L = 1$ and parity = No change: the multipole transition is $M1$

Ans. 2: (a)

Solution: $L_0 = mvr_\perp$



At the highest point,

$$v = \text{speed } = v_0 \cos 30^0 = \frac{\sqrt{3}v_0}{2}$$

$$r_\perp = h = \frac{v_0^2 \sin^2 30^0}{2g} = \frac{v_0^2}{8g}$$

$$\Rightarrow \quad L_0 = m\left(\frac{\sqrt{3}v_0}{2}\right)\left(\frac{v_0^2}{8g}\right) = \frac{\sqrt{3}mv_0^3}{16g}$$

Ans. 3: (a)

Solution: In this case; $E_{in} = 0, E_{out} \neq 0$. So $E_{out} > E_{in}$

Ans. 4: (b)

Solution: The electron configuration of Mn^{3+} is $[Ar]3d^4$

$M_L = -2 \ \ -10 \ \ 0 \ \ +1 \ \ +2$

For d^4:

↑	↑	↑	↑	

$\therefore$ The highest $S = 2$

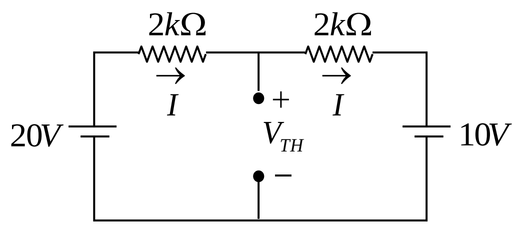
and the highest $L = 3$

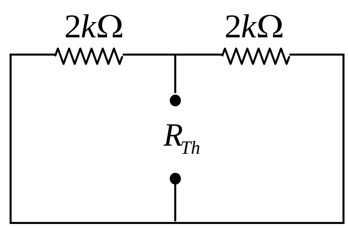
$\therefore \quad J = 1, 2, 3, 4, 5$

The ground state is $^{2S+1}L_J = {}^5F_1$

Ans. 5: (a)

Solution:





$$-20+2I+2I+10=0 \Rightarrow I=\frac{5}{2}A$$

$$R_{TH}=\frac{2\times 2}{2+2}=1k\Omega$$

$$-20+2\times\frac{5}{2}+V_{Th}=0 \Rightarrow V_{Th}=15V$$

Ans. 6: 3.5

Solution: $\langle H\rangle=\frac{5\times 1}{1+4+9}+\frac{2\times 4}{1+4+9}+\frac{4\times 9}{1+4+9}=\frac{5}{14}+\frac{8}{14}+\frac{36}{14}=\frac{49}{14}=3.5$

Ans. 7: (b)

Solution: We have $\frac{dx}{dt}=ky$ and $\frac{dy}{dt}=kx$

Hence $\frac{dy}{dx}=\frac{x}{y} \Rightarrow \int ydy=\int xdx+c \Rightarrow y^2=x^2+c \Rightarrow y^2-x^2=c$

Ans. 8: 1.25

Solution: $\varepsilon_0=2\times\frac{\pi^2\hbar^2}{2mL^2}$

$$E=2\times\frac{\pi^2\hbar^2}{2m(2L)^2}+2\times\frac{4.\pi^2\hbar^2}{2m(2L)^2}=2\times\frac{\pi^2\hbar^2}{2mL^2}\left(\frac{1}{4}+\frac{4}{4}\right)=\varepsilon_0\frac{5}{4}=1.25\varepsilon_0$$

Ans. 9: (d)

Solution: Let $\vec{F}=F_0(\hat{x}+\hat{y}+\hat{z})$ and $\vec{r}=x\hat{x}+y\hat{y}+z\hat{z} \Rightarrow \vec{F}.\vec{r}=F_0(x+y+z)$.

Thus $\vec{\nabla}(\vec{F}\cdot\vec{r})=F_0(\hat{x}+\hat{y}+\hat{z})=\vec{F} \Rightarrow |\vec{r}|\vec{\nabla}(\vec{F}\cdot\vec{r})=|\vec{r}|\vec{F}$

Ans. 10: (a)

Solution: In KCl crystal only even (hkl) planes are present. All odd (hkl) and mixed (hkl) are absent. Thus, correct option is (a)

Ans. 11: 5.3

Solution: Energy is conserved.

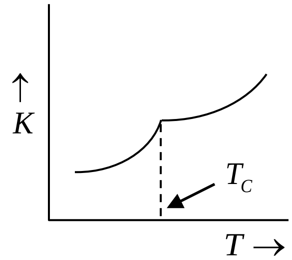
Loss in kinetic energy = Gain in potential energy

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_{\min}} = 5\times\left(1.6\times10^{-13}\right)J \Rightarrow r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5\times1.6\times10^{-13}}$$

$$\text{or } r_{\min} = \frac{\left(9\times10^9\right)(2)(92)\left(1.6\times10^{-19}\right)^2}{5\times1.6\times10^{-13}} \Rightarrow r_{\min} = 5.3\times10^{-14}m$$

Ans. 12: (c)

Solution: The coefficient of thermal conductivity is continuous at the critical temperature.



Ans. 13: (d)

Solution: $z = 29\ :\ (1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^2(1d_{5/2})^6(1s_{1/2})^2(1d_{3/2})^6(1f_{7/2})^4$

$$\therefore I = \frac{7}{2} \text{ and } P = (-1)^l = (-1)^3 = -v_e$$

$$\therefore \text{spin} = \left(\frac{7}{2}\right)^-$$

Ans. 14: 2.45

Solution: Intensity, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$, where E_0 is amplitude of the electric field of the light.

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$E_0 = \sqrt{\frac{2P}{4\pi r^2 c\varepsilon_0}} = 2.45V/m$$

Ans. 15: (b)

Solution: Stopping potential is the negative potential which stops the emission of $(K.E)_{\max}$ electrons when applied.

$\therefore$ Stopping potential = 4 volt

Ans. 16: 1.81

Solution: $n = z\dfrac{\rho N_A}{M} = \dfrac{3\times 2.7\times 6.02\times 10^{23}}{27} cm^{-3} = 1.81\times 10^{23} cm^{-3} = 1.81\times 10^{29} m^{-3}$

Ans. 17: (b)

Solution: $P(0) = \dfrac{1}{1+e^{-\varepsilon / k_B T}}$

Population with energy 0

$$= N \cdot \frac{1}{1+e^{-\varepsilon / k_B T}}$$

$\lim \varepsilon / k_B T \ll 1$

$$= \frac{N}{2}$$

Ans. 18: 4

Solution: $\left|\vec{E}\right| \times 2\pi r = -\dfrac{\partial B}{\partial t} \times \pi r^2 \Rightarrow \left|\vec{E}\right| = \dfrac{r}{2}\dfrac{\partial B}{\partial t} = \dfrac{2\times 10^{-2}}{2} 0.4 = 4\ mV/m$

Ans. 19: (a)

Ans. 20: 0.24

Solution: Smallest voltage step $= \dfrac{1}{2^{12}-1} \approx 0.24\ mV$

Ans. 21: (a)

Solution: $n = 3, l = 2, m = 1$

Ans. 22: 3.6

Solution: $R = R_D A^{1/3}$

$$\therefore \frac{R_{Mg}}{R_{Cu}} = \left[\frac{A_{Mg}}{A_{Cu}}\right]^{1/3} = R_{Mg} = R_{Cu}\left(\frac{A_{Mg}}{A_{Cu}}\right)^{1/3}$$

$$\therefore R_{Mg} = 4.8\times 10^{-15} m \left(\frac{27}{64}\right)^{1/3} = 3.6\times 10^{-15} m = 3.6\ fm$$

Ans. 23: (d)

Solution: For electric dipole transition, $L = 2$ and parity remain unchanged.

Thus $O^{+} \rightarrow 2^{+}$ is through $E2$ transition.

Ans. 24: (b)

Solution: This matrix is Skew Hermitian Matrix

$$\begin{vmatrix} -\lambda & 0 & i \\ 0 & i-\lambda & 0 \\ i & 0 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda\left[-\lambda(i-\lambda)-0\right]+i\left[0-i(i-\lambda)\right]=0$$

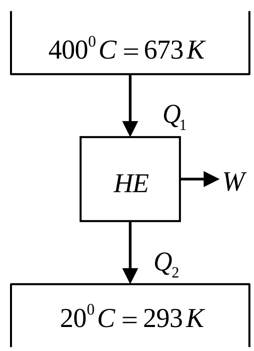
$$\Rightarrow \lambda^{2}(i-\lambda)+(i-\lambda)=0$$

$$\Rightarrow (i-\lambda)(\lambda^{2}+1)=0 \Rightarrow \lambda_1 = i,\ \lambda_2 = i \text{ and } \lambda_3 = -i$$

Thus matrix D is $\begin{bmatrix} i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & -i \end{bmatrix}$. Hence $e^{\pi D} = \begin{bmatrix} e^{i\pi} & 0 & 0 \\ 0 & e^{i\pi} & 0 \\ 0 & 0 & e^{-i\pi} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

Ans. 25: 0.56

Solution: Since working fluid evaporates at $400^{0}C$ and entropy change $1.5\,kJ/K$ so heat supplied per kg.



$$Q_1 = 1.5 \times 673 = 1009.5\,kJ/kg$$

Efficiency of heat engine

$$\eta = 1 - \frac{293}{673} = 0.57$$

$$W = \eta Q_1 = 0.57 \times 1009.5 = 570\,kJ/kg.$$

Ans. 26: (b)

Solution: $\varepsilon_u = \omega_e\left(v+\frac{1}{2}\right) - \omega_e x_e\left(v+\frac{1}{2}\right)^{2}$

$$\frac{d\varepsilon}{dv} = 0 \Rightarrow \omega_e - 2\omega_e x_e\left(v+\frac{1}{2}\right) = 0 \Rightarrow v_{\max} = \frac{1}{2x_e} - \frac{1}{2}$$

$$\therefore \Delta\varepsilon = \varepsilon_{v_{\max}} - \varepsilon_0 = \left[\omega_e\left(v_{\max}+\frac{1}{2}\right) - \omega_e x_e\left(v_{\max}+\frac{1}{2}\right)^{2}\right] - \left[\frac{\omega_e}{2} - \frac{\omega_e x_e}{4}\right]$$

$$= \left[\omega_e \frac{1}{2x_e} - \omega_e x_e \frac{1}{4x_e^{2}}\right] - \left[\frac{\omega_e}{2} - \frac{\omega_e x_e}{4}\right]$$

$$= \frac{\omega_e}{4x_e} - \frac{\omega_e}{2} + \frac{\omega_e x_e}{4} = \omega_e\left(\frac{2}{x_e} - \frac{1}{2}\right) = \omega_e v_{\max}$$

$\therefore$ The dissociation energy is $E_0 = hc \cdot \omega_e v_{\max}$

where $\omega_e = 2990\,cm^{-1}$ and $\frac{2\omega_e(1-3x_e)}{\omega_e(1-2x_e)} = 1.96\,cm^{-1} \Rightarrow 1.04\,x_e = 0.02$

$$\Rightarrow x_e = 0.0192$$

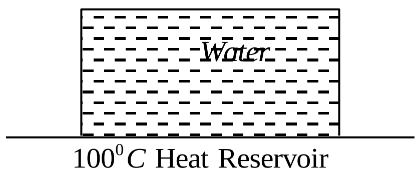
$$\therefore E_D = 1.239\times10^{-4}\,eV - cm \times 2990\,cm^{-1} \times 0.0192$$

$$= 7.124\times10^{-3}\,eV = 7.12\,meV$$

Ans. 27: 0.186

Solution: Entropy change of water,

$$\Delta s_w = \int_{273}^{373} \frac{mc_p dT}{T} = 1\times 4.2 \ln\frac{373}{273} = +1.316\ kJ/K$$



Entropy change of reservoir,

$$\Delta s_{res} = -\frac{Q}{T} = \frac{mc_p(100-0)}{373} = -\frac{1\times4.2\times100}{373} = -1.13\,kJ/K$$

$$\Delta s_{univ} = \Delta s_{total} = s_{gen} = 1.316 - 1.13 = 0.186\,kJ/K$$

Ans. 28: (a)

Solution: $\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q} = 1$ for $P = \log\sin p$ and $P = q\tan p$

$\frac{\partial Q}{\partial q}.\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}.\frac{\partial P}{\partial q} \neq 1$ for $P = q^2 \sin 2p$ and $Q = q^2 \cos 2p$

Ans. 29: 0.04

Solution: $I = \frac{1}{2\pi i}\oint z^7 \cos\left(\frac{1}{z^2}\right)dz$

Finding the residue

$$z^7 \cos\left(\frac{1}{z^2}\right) = z^7\left[1 - \left(\frac{1}{z^2}\right)^2 \frac{1}{2!} + \left(\frac{1}{z^2}\right)^4 \frac{1}{4!}\ldots\right] = z^7\left[1 - \frac{1}{z^4}\frac{1}{2!} + \frac{1}{z^8}\frac{1}{4!}\ldots\right]$$

$$= z^7 - z^3\frac{1}{2!} + \frac{1}{z}\left(\frac{1}{4!}\right)$$

$\therefore \operatorname*{Res}_{x=0} f(z) = \frac{1}{4!}$

$\therefore I = \frac{1}{2\pi i}\left[2\pi i \times \frac{1}{4!}\right] = \frac{1}{24} = 0.041$

Hence, Answer is 0.041

Ans. 30: (a)

Solution: $\Delta\lambda = \frac{\lambda^2}{c}\cdot\frac{eB}{4\pi m} = \frac{\lambda^2}{c}\cdot\mu_B\cdot\frac{B}{h}$

$= \frac{\left(650\times10^{-9} m\right)^2}{3\times10^8 m/s}\times1.53\times10^{-24}\ J/T\cdot\frac{2T}{6.625\times10^{-34}\ J\cdot s} = 6.5\times10^{-12} m = 6.5\ pm$

Ans. 31: (c)

Solution: $B_A = \frac{\mu_0}{2}\frac{i}{(l/2\pi)}$ and $B_B = \left[\frac{\mu_0}{4\pi}\frac{i}{l/8}\left(\sin 45° + \sin 45°\right)\right]\times 4$

$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$

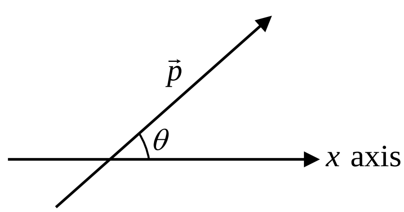
Ans. 32: 60

Solution: So from $\vec{\tau} = \vec{p}\times\vec{E}$

$\tau\hat{k} = \left(p_x\hat{i} + p_y\hat{j}\right)\times\left(E\hat{i}\right) = -p_yE\hat{k}$

$-\tau\hat{k} = \left(p_x\hat{i} + p_y\hat{j}\right)\times\left(\sqrt{3}E\hat{j}\right) = p_x\times\sqrt{3}E\hat{k}$

Thus $-\left(-p_yE\hat{k}\right) = p_x\times\sqrt{3}E\hat{k}$



$\Rightarrow \frac{p_y}{p_x} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} \quad \Rightarrow \theta = 60^0$

Ans. 33: (d)

Solution: $L\{\sin\beta t + \beta t\cos\beta t\} = \frac{\beta}{\left(s^2+\beta^2\right)} - \beta\frac{d}{ds}\left[\frac{s}{\left(s^2+\beta^2\right)}\right]$

$= \frac{\beta}{s^2+\beta^2} - \beta\frac{\left(\beta^2 - s^2\right)}{\left(s^2+\beta^2\right)^2} = \frac{\beta\left(s^2+\beta^2\right) - \beta\left(\beta^2 - s^2\right)}{\left(s^2+\beta^2\right)^2} = \frac{2\beta s^2}{\left(s^2+\beta^2\right)^2}$

Ans. 34: (b)

Solution: $\frac{dU}{dx}=0 \Rightarrow k\left(6x^2-10x+4\right)=0 \Rightarrow x=1, x=\frac{2}{3}$

$$\frac{d^2U}{dx^2}=k\left(12x-10\right)$$

For $x=1$, $\frac{d^2U}{dx^2}=2k>0$ and for $x=\frac{2}{3}$, $\frac{d^2U}{dx^2}=-4k<0$ So

$$\omega=\sqrt{\frac{2k}{m}} \Rightarrow \frac{2\pi}{T}=\sqrt{\frac{2k}{m}} \Rightarrow T=2\pi\sqrt{\frac{m}{2k}} \Rightarrow T=\pi\sqrt{\frac{2m}{k}}$$

Ans. 35: 0.55

Solution: $x_{cm}=\frac{\int_0^L x\,dm}{\int_0^L dm}=\frac{\int_0^L x\rho\,dx}{\int_0^L \rho\,dx}=\frac{\int_0^L x\rho_0\left(1+\frac{x}{L}\right)dx}{\int_0^L \rho_0\left(1+\frac{x}{L}\right)dx}=\frac{\left[\frac{x^2}{2}+\frac{x^3}{3L}\right]_0^L}{\left[x+\frac{x^2}{2L}\right]_0^L}=\frac{\frac{L^2}{2}+\frac{L^3}{3L}}{L+\frac{L^2}{2L}}=\frac{L^2\left(\frac{1}{2}+\frac{1}{3}\right)}{L\left(1+\frac{1}{2}\right)}$

$$=\frac{5L/6}{3/2}=\frac{5}{9}L=0.55L$$

Ans. 36: (d)

Solution: (a) At triple point, all the phase are in equilibrium so,

$$\ln\ p \Rightarrow 15.16-\frac{3063}{T}=18.7-\frac{3754}{T}$$

$$\frac{\left(3754-3063\right)}{T}=18.7-15.16=3.54$$

$$\frac{691}{T}=3.54 \Rightarrow T=\frac{691}{3.54}=195.2K$$

Put this value of T in any of two given equations,

$$\ln\ p=15.16-\frac{3063}{195.2}=15.16-15.6918=0.53178$$

$p=0.58756$ atmosphere $\quad =59.53\ kPa$.

Ans. 37: 1.25

Solution: The condition for constructive interference is

$$2\mu t=\left(m+\frac{1}{2}\right)\lambda\text{, where } m=0,1,2$$

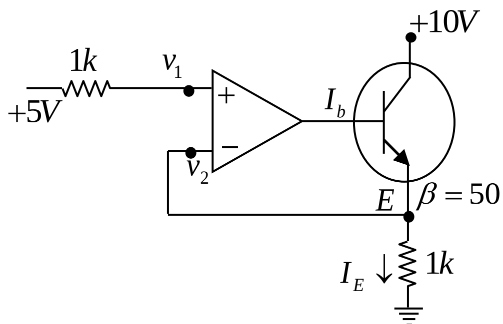
$\therefore$ for $m=0$; $\quad \mu = \dfrac{\lambda/2}{2t} = \dfrac{\lambda}{4t} = \dfrac{0.5\times10^{-6}m}{4\times0.1\times10^{-6}m} = \dfrac{5}{4} = 1.25$

Ans. 38: (d)

Solution: For constant velocity $q\left[\vec{E}+\left(\vec{V}\times\vec{B}\right)\right]=0$

Ans. 39: 0.1

Solution: $v_2 = 5V = V_E$



$$I_E = \frac{5}{1} = 5\,mA$$

$$\beta I_B = 5\,mA \Rightarrow I_B = \frac{5}{50}mA = 0.1\ mA$$

Ans. 40: 0.19

Solution: $\dfrac{mv_0^2}{r} = \dfrac{GMm}{r^2} \Rightarrow mv_0^2 = \dfrac{GMm}{r}$

Resultant velocity at the each fragment from earth $v = \sqrt{v_0^2 + \dfrac{v_0^2}{4}} = \dfrac{v_0\sqrt{5}}{2}$

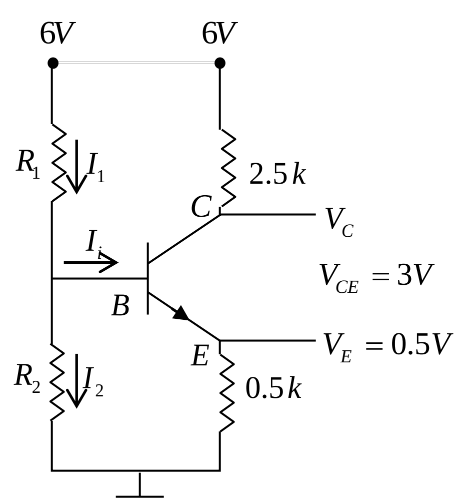
So kinetic energy $T = \dfrac{1}{2}\dfrac{m}{2}v^2 = \dfrac{5}{16}mv_0^2 = \dfrac{5GmM}{16r}$

Potential energy is $-\dfrac{GmM}{2r}$

Total energy $E = T + V = -\dfrac{3GMm}{16r} = -0.1875\dfrac{GMm}{r}$

Ans. 41: (b)

Solution: $I_1 = I_i + I_2 \approx I_2 = 10I_B$, $\quad I_C = 1mA,\ \beta = 50, V_{BE} = 0.7V$



$$V_B = I_2R_2 = V_{BE} + I_ER_E = 0.7 + 0.5 = 1.2\ V$$

$$\Rightarrow R_2 = \frac{1.2\ V}{I_2} = \frac{1.2\ V}{10I_B} = \frac{1.2\ V}{10\times(1/50)} = 6\ k\Omega$$

$$\because V_B = \frac{V_{CC}R_2}{R_1+R_2} \Rightarrow 1.2 = \frac{6\times6}{R_1+6} \Rightarrow R_1 = 24\ k\Omega$$

$$\Rightarrow \frac{R_2}{R_1} = \frac{6}{24} = 0.25$$

Ans. 42: 4

Solution: $n_i = \sqrt{N_c N_v} e^{-E_g/2kT} \Rightarrow \frac{n_{i1}}{n_{i2}} = \frac{e^{-E_g/2kT_1}}{e^{-E_g/2kT_2}} \Rightarrow \frac{10^{10}\, cm^{-3}}{n_{i2}} = \exp\left[-\frac{E_g}{2k}\left(\frac{1}{300} - \frac{1}{200}\right)\right]$

$$\Rightarrow n_{i2} = 10^{10} \times \exp\left[\frac{E_g}{2k}\left(\frac{1}{300} - \frac{1}{200}\right)\right] = 10^{10} \times \exp\left[\frac{1.25 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 100}\left(-\frac{1}{6}\right)\right]$$

$$= 10^{10} \times 5.7 \times 10^{-6} \Rightarrow n_{i2} = 5.7 \times 10^{4}\, cm^{-3} \Rightarrow N = 4$$

Ans. 43: (c)

Solution: For first exited state, $|\phi_{2,1}\rangle = \frac{2}{a}\sin\frac{2\pi x}{a}\sin\frac{\pi y}{a}$, $|\phi_{1,2}\rangle = \frac{2}{a}\sin\frac{\pi x}{a}\sin\frac{2\pi y}{a}$

$H_0 = \begin{bmatrix} 5E_0 & 0 \\ 0 & 5E_0 \end{bmatrix}$ $\quad H_P = \begin{bmatrix} V_0 & 0 \\ 0 & V_0 \end{bmatrix}$ Where $E_0 = \frac{\pi^2 \hbar^2}{2ma^2}$

$H = H_0 + H_P = \begin{bmatrix} 5E_0 + V_0 & 0 \\ 0 & 5E_0 + V_0 \end{bmatrix}$ So Eigen value is $5E_0 + V_0$, $5E_0 + V_0$

Ans. 44: (b)

Solution: (a) $\pi^+ + p \to \Sigma^+ + k^+$

	π^+	$+p$	Σ^+	$+k^+$	
q:	$+1$	$+1$	$+1$	$+1$	: Conserved
spin:	0	$\frac{1}{2}$	$\frac{1}{2}$	0	: Conserved
B:	0	$+1$	$+1$	0	: Conserved
I:	1	$\frac{1}{2}$	1	$\frac{1}{2}$	: Conserved
I_3:	$+1$	$\frac{1}{2}$	$+1$	$\frac{1}{2}$	: Conserved
S:	0	0	-1	$+1$	: Conserved

It is allowed.

(b) $\Omega^- \to \Lambda^0 + k^-$

q: -1 $\quad 0 \quad -1$: Conserved

Spin: $\frac{3}{2}$ $\quad \frac{1}{2}$ $\quad 0$: Not conserved

This is not allowed interaction.

Ans. 45: 18.04

Solution: Kinetic energy of Auger electron = kinetic energy of electron in k - shell – 2 (Kinetic energy of electron in L - shell)

$= 28.5 - 2\times 5.23 = 18.04\, keV$

Ans. 46: 10

Solution: Given $u(x,y) = x^3 + ax^2y + bxy^2 + 2y^3$

$u_x = 3x^2 + 2axy + by^2, u_{xx} = 6x + 2ay$

$u_y = ax^2 + 2bxy + 6y^2, u_{yy} = 2bx + 12y$

As, u is harmonic, therefore $u_{xx} + u_{yy} = 0$

$\Rightarrow 6x + 2ay + 2bx + 12y = 0 \Rightarrow (6+2b)x + (2a+12)y = 0$

$\Rightarrow 6 + 2b = 0 \Rightarrow b = -3$ and $2a + 12 = 0$

$\Rightarrow 2a = -12 \Rightarrow a = -6$

As, $v(x,y)$ is harmonic conjugate of $u(x,y)$

Therefore, $u_x = v_y$ & $u_y = -v_x$

$\Rightarrow v_y = 3x^2 - 12xy - 3y^2$

Integrating w.r.t x, we get

$\Rightarrow v = 3x^2y - 12x\dfrac{y^2}{2} - \dfrac{3y^3}{3} + \phi(x) \Rightarrow v = 3x^2y - 6xy^2 - y^3 + \phi(x)$ ...(i)

Differentiate w.r.t. x, we get $v_x = 6xy - 6y^2\phi'(x)$ ...(ii)

Also, $v_x = 6x^3 + 6xy - 6y^2$

Comparing (i) and (ii), we get $\phi'(x) = 6x^2 + c$

On integrating, we $\phi(x) = 2x^3 + c$

Now given $v(0,0) = 1 \Rightarrow c = 1$

$\therefore v(x,y) = 3x^2y - 6xy^2 - y^3 + 2x^3 + c$

Now given $v(0,0) = 1 \Rightarrow c = 1$

$\therefore \phi(x) = 2x^3 + 1,$

$\therefore v = 3x^2y - 6xy^2 - y^3 + 2x^3 + 1$

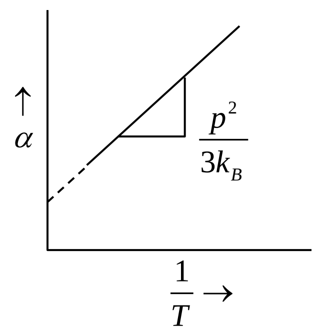
$v(1,1) = 3 - 6 - 1 + 2 + 1 = -7 + 6 = -1$

$|a + b + v(1,1)| = |-6 - 3 - 1| = |-10| = 10$

Hence, Answer is 10.

Ans. 47: (d)

Solution: In graph between α and $1/T$, the slope represent the electric dipole moment. Therefore,



$(slope)_D > (slope)_B > (slope)_A > (slope)_C$

Thus, $p_D > p_B > p_A > p_C$.

Correct option is (d)

Ans. 48: (c)

Solution: $|\psi\rangle = \cos\left(\frac{\theta}{2}\right)e^{-i\varphi}\left|\frac{1}{2}, \frac{1}{2}\right\rangle + \sin\left(\frac{\theta}{2}\right)\left|\frac{1}{2}, -\frac{1}{2}\right\rangle \Rightarrow \begin{pmatrix} \cos\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2} \end{pmatrix}$

$$\langle\psi| = \begin{pmatrix} e^{i\phi}\cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{pmatrix}$$

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\langle S_x\rangle = \langle\psi|S_x|\psi\rangle = \begin{pmatrix} e^{i\phi}\cos\frac{\theta}{2} \cdot \sin\frac{\theta}{2} \end{pmatrix}\frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \frac{\hbar}{2}\begin{pmatrix} \sin\frac{\theta}{2} & e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2}e^{-i\phi} \\ \sin\frac{\theta}{2} \end{pmatrix} = \frac{\hbar}{2}\left(\sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{-i\phi} + \sin\frac{\theta}{2}\cos\frac{\theta}{2}e^{i\phi}\right)$$

$$= \frac{\hbar}{2} \cdot 2 \cdot \left(\sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}\right)\cos\phi = \frac{\hbar}{2}\sin\theta \cdot \cos\phi$$

Ans. 49: 1.43

Solution: $E^2 = p^2c^2 + \left(m_0c^2\right)^2 \Rightarrow p = \sqrt{\frac{E^2 - \left(m_0c^2\right)^2}{c^2}} = \frac{\sqrt{1-0.25}}{c} = \frac{\sqrt{0.75}MeV}{c}$

$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\times3\times10^{8}}{\sqrt{0.75}\times1.6\times10^{-13}} = \frac{19.8\times10^{-13}}{1.38} = 14.34\times10^{-13}m = 1.43\times10^{-12}m$$

Ans. 50: (c)

Solution: Total number of atoms in solid $= pN$

Total degree of freedom $= 3pN$

Acoustical modes $= 3N$

Optical Modes $3pN - 3N$

Thus, correct option is (c)

Ans. 51: 0.474

Solution: $Q_{\beta^+} = \left[M\left(Z,A\right) - M\left(Z-1,A\right) - 2m_e\right]c^2$

$$= \left[M\left(Z,A\right) - M\left(Z-1,A\right)\right]c^2 - 2m_ec^2$$

$$= \left[39.96399 - 39.962384\right]931.5 - 1.022 = 0.474\,MeV$$

Ans. 52: 0.89

Solution: Radius of Fermi surface is

$$K_F = \left(\frac{2\pi N}{A}\right)^{1/2}$$

Where, $\frac{N}{A} = \frac{N}{\text{Area of unit cell}} = \frac{1}{8\times10^{-20}m^2} = 1.25\times10^{19}m^{-2}$

$$\therefore K_F = \left(2\pi\times1.25\times10^{19}\right)^{1/2} = \left(7.85\times10^{19}\right)^{1/2}$$

$$\approx 0.886\left(\overset{0}{\mathrm{A}}\right)^{-1} = 0.889\left(\overset{0}{\mathrm{A}}\right)^{-1}$$

Ans. 53: 0.75

Solution: $E = \mu.\frac{s^2 - s_1^2 - s_2^2}{2}$ for triplet state. If $s_1 = \frac{1}{2}$ $s_2 = \frac{1}{2}$, then $s = 1$

$$E = \mu\frac{3\hbar^2 - \frac{3\hbar^2}{4} - \frac{3\hbar^2}{4}}{2} = \frac{\left(12-3-3\right)\mu\hbar^2}{8} = \frac{6}{8}\mu\hbar^2 = \frac{3}{4}\mu\hbar^2 = 0.75\mu\hbar^2$$

Ans. 54: (b)

Solution: $E = -\vec{\mu}\cdot\vec{H} = \sum_{\theta_i} -\mu H \cos\theta_i$

$$Z_1 = \int_0^{2\pi}\int_0^{\pi} e^{\frac{\mu H\cos\theta}{kT}} \sin\theta \, d\theta \, dp = 2\pi \int_0^{\pi} \sin\theta \, e^{\frac{\mu H\cos\theta}{kT}} d\theta$$

$$= 2\pi\left(-\frac{kT}{\mu H}\right)\int_{\frac{\mu H}{kT}}^{-\mu H/kT} e^x dx = -\frac{2\pi kT}{\mu H}\left[e^{-\mu H/kT} - e^{\mu H/kT}\right]$$

$$= \frac{2\pi kT}{\mu H}\left(e^{\mu H/kT} - e^{-\mu H/kT}\right) = \frac{4\pi kT}{\mu H}\sinh\left(\frac{\mu H}{kT}\right) = 4\pi\frac{\sinh(\mu H/kT)}{(\mu H/kT)},$$

$$Z_n = Z_1^N$$

$$Z_n = (4\pi)^N\left(\frac{\sinh(\mu H/kT)}{(\mu H/kT)}\right)^N$$

Ans. 55: 1.66

Solution: Speed of cosmic particle with respect to spaceship

$$u_x = \frac{u_x + v}{1+\frac{u_x v}{c^2}} = \frac{0.5c + 0.5c}{1+\frac{(0.5)\times(0.5)c^2}{c^2}} = \frac{c}{1+0.25} = \frac{4c}{5}$$

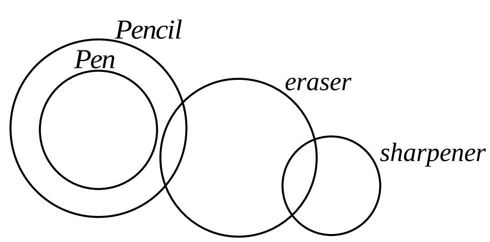
$$u_y' = 0, \qquad u = \sqrt{u_x'^2 + u_y'^2 + u_y'^2} = \frac{4c}{5}, m = \frac{m_O}{\sqrt{1-\frac{u^2}{c^2}}} = \frac{5m_O}{3} = 1.66m_0$$

Ans. 56: (c)

Ans. 57: (c)

Ans. 58: (d)

Solution: From the venn- diagram it is obvious that option (d) is correct in the stated conditions.



Ans. 59: (c)

Solution: Let volume of cylinder $= \pi r^2 h$

Where $r=$ radius and $h=$ Height

New radius $=r\left(1+\frac{50}{100}\right)=\frac{3r}{2}$

Increase in volume $=\pi\left(\frac{3r}{2}\right)^2 h-\pi r^2 h=\frac{5}{4}\pi r^2 h$

So, % increases in volume $=\frac{\frac{5}{4}\pi r^2 h}{\pi r^2 h}\times 100$

$=125$.

Ans. 60: (d)

Let, Median $=y$

Mean $=\frac{1+x+x+x+y+y+11+15+17}{9}=\frac{3x+2y+44}{9}$

Mode $=x$.

Given, $\frac{3x+2y+44}{9}=y=2x$

Solving, we get $x=4$ and $y=8$.

Ans. 61: (b)

Ans. 62: (b)

Solution: Following the allotment order and construction we get the following allocation list.

$A\rightarrow Z$ $\quad C\rightarrow X$

$B\rightarrow W$ $\quad D\rightarrow Y$

So, correct choice is (b)

Ans. 63: (c)

Solution: $(3181)^7$ will end in $1^7=\_1$

$(3182)^9$ will end in $2^9=\_2$

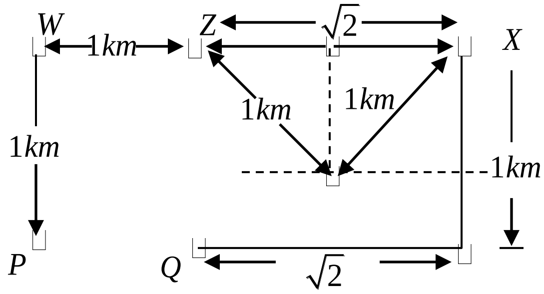
$(3183)^{11}$ will end in $3^9=\_7$

$(3184)^{13}$ will end in $4^{13}=\_4$

So, required sum will have digit $=1+2+7+4=14$.

Ans. 64: (c)

Solution:



Location of each has been given in fig.

So, distance between

X and $Q = \sqrt{\left(\sqrt{2}\right)^2 + (1)^2} = \sqrt{3}$

Correct choice (c).

Ans. 65: (d)

Solution: (a) Position: $A \quad L \quad R \quad V \quad X$

$1 \quad 12 \quad 18 \quad 22 \quad 24$

Difference: $11 \quad 6 \quad 4 \quad 2$

(b) Position: $E \quad P \quad V \quad Z \quad B$

$5 \quad 16 \quad 22 \quad 26 \quad 2$

Difference: $11 \quad 6 \quad 4 \quad 2$

(c) Position: $I \quad T \quad Z \quad D \quad F$

$9 \quad 20 \quad 26 \quad 4 \quad 6$

Difference: $11 \quad 6 \quad 4 \quad 2$

(d) Position: $O \quad Y \quad E \quad I \quad K$

$15 \quad 25 \quad 5 \quad 9 \quad 11$

Difference: $10 \quad 6 \quad 4 \quad 2$
