ALL INDIA TEST SERIES

IIT - JAM - 2019

FULL LENGTH TEST – 01

20-01-2019

PHYSICS (PH)

TIME: 3 HOURS **MAXIMUM MARKS: 100**

Section A: This section contains a total of 30 Multiple Choice Questions **(MCQ)** carrying one or two marks each. Each MCQ type question has four choices out of which only one choice is the correct answer.

There will be negative marking @ $\frac{1}{3}^{rd}$ for one marks MCQ and $\frac{2}{3}^{rd}$ negative marks for two marks MCQ for each wrong answer.

Section B: This section contains a total of 10 Multiple Select Questions **(MSQ)** carrying two marks each. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers.

Section C: This section contains a total of 20 Numerical Answer Type **(NAT)** questions carrying one or two marks each. For these NAT type questions, the answer is a signed real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these types of questions.

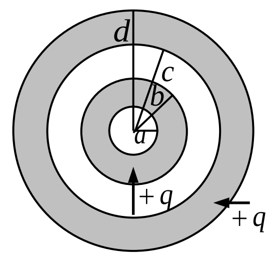
Note: There will be no negative marking for Section B and Section C.

SECTION A

Multiple Choice Questions (MCQ)

Q1 – Q10 Carry One Mark each. (1/3 negative marks for each wrong answer)

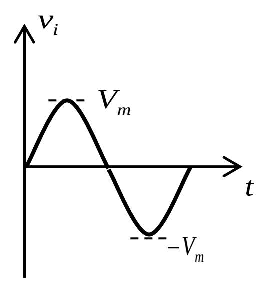
Q1. Consider two concentric conducting spherical shells with inner and outer radii a, b and c, d as shown in the figure. Both the shells are given q amount of positive charges. In order to have equal surface charge densities on the outer surface of both the shells, the following conditions should be satisfied

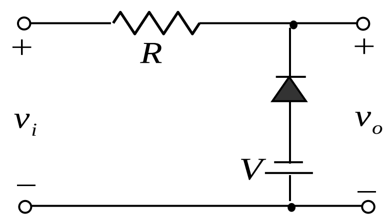


(a) $d = 4b$ and $c = 2a$ (b) $d = 2b$ and $c = \sqrt{2}a$

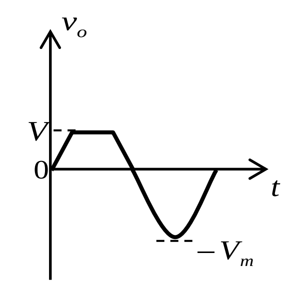
(c) $d = \sqrt{2}b$ and $c > a$ (d) $d > b$ and $c = \sqrt{2}a$

Q2. Identify the correct output waveforms for the circuit given below (assume diodes to be ideal)





(a)



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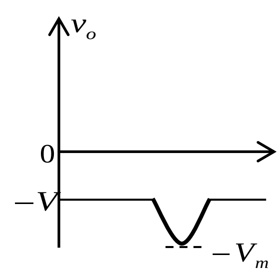

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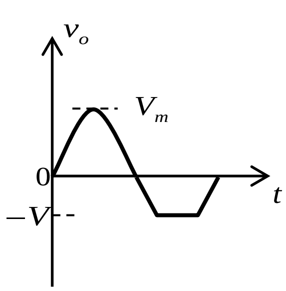

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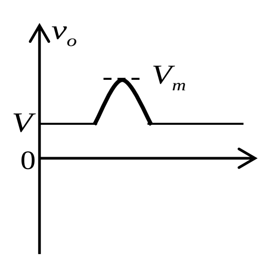


Q3. The unit normal vector of the point $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$ on the surface of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

(a) $\frac{bc\hat{i}+ca\hat{j}+ab\hat{k}}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$ (b) $\frac{a\hat{i}+b\hat{j}+c\hat{k}}{\sqrt{a^2+b^2+c^2}}$

(c) $\frac{b\hat{i}+c\hat{j}+a\hat{k}}{\sqrt{a^2+b^2+c^2}}$ (d) $\frac{b^2c^2\hat{i}+c^2a^2\hat{j}+a^2b^2\hat{k}}{\sqrt{b^4c^4+a^4c^4+a^4b^4}}$

Q4. The speed of a transverse wave on a stretched string is $500\,m/s$, when it is stretched under a tension of $19.6\,N$. If the tension is altered to a value of $78.4\,N$, the speed of the wave is

(a) $950\,m/s$ (b) $975\,m/s$ (c) $1000\,m/s$ (d) $1015\,m/s$

Q5. The Q - value of a tuning fork of frequency $512\,Hz$ is given as 8×10^4. The time in which the energy is reduced by $\frac{1}{e}$ of its initial value is

(a) $24.9\sec$ (b) $26.8\sec$ (c) $27.5\sec$ (d) $28.2\sec$

Q6. A particle of mass m is projected with a velocity v_0 making an angle of 30^0 with the horizontal. The magnitude of the angular momentum of the projectile about the point of projection when the particle is at its maximum height h is:

(a) $\frac{\sqrt{3}mv_0^3}{16g}$ (b) $mv_0^3/\left(4\sqrt{2}g\right)$ (c) $mv_0^3/\left(\sqrt{2}g\right)$ (d) $m\sqrt{2gh^3}$

Q7. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy $10\,eV$ fall on it is $4\,eV$. The stopping potential in volt is

(a) 6 (b) 4 (c) 8 (d) 10

Q8. In YDSE, the separation between the slits is halved and the distance between the slits and the screen is doubled. The fringe width is

(a) Unchanged (b) Halved (c) Doubled (d) Quadrupled

Q9. If $y' = \frac{dy}{dx}$ then the solution of the differential equation $y' + y\tanh x = 2e^x$ is (c is an arbitrary constant)

(a) $y = \frac{e^{2x} + e^x + c}{e^x + e^{-x}}$

(b) $y = \frac{e^{2x} + e^{-x} + c}{e^x + e^{-x}}$

(c) $y = \frac{e^{2x} - 2x + c}{e^x + e^{-x}}$

(d) $y = \frac{e^{2x} + 2x + c}{e^x + e^{-x}}$

Q10. If $z = \cos\left(\log i^i\right)$ then the value of z^2 is

(a) 0 (b) 1 (c) 2 (d) 3

Q11 – Q30 Carry Two Marks each (2/3 negative marks for each wrong answer)

Q11. Two identical wires A and B each of length $'l'$, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side $'a'$. If B_A and B_B are the values of magnetic field at the centers of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(a) $\frac{\pi}{16\sqrt{2}}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{8\sqrt{2}}$ (d) $\frac{\pi^2}{8}$

Q12. A charged particle (proton) is introduced at the origin $(x=0, y=0, z=0)$ with a given initial velocity $\vec{v}$. In which case will the particle move in a straight line with constant velocity? (The quantities E_0, B_0 are positive in magnitude)

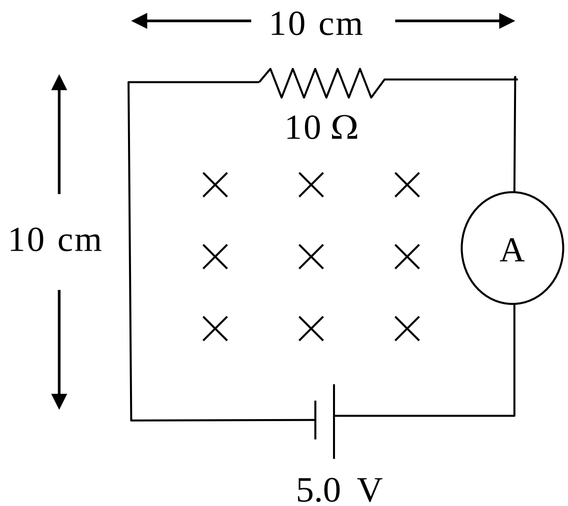
(a) $\vec{v} = \frac{E_0}{B_0}\hat{y}$, $\vec{E} = -E_0\hat{y}$, $\vec{B} = B_0\hat{z}$

(b) $\vec{v} = \frac{E_0}{B_0}\hat{y}$, $\vec{E} = -E_0\hat{x}$, $\vec{B} = B_0\hat{y}$

(c) $\vec{v} = 2\frac{E_0}{B_0}\hat{y}$, $\vec{E} = -E_0\hat{x}$, $\vec{B} = B_0\hat{z}$

(d) $\vec{v} = \frac{E_0}{B_0}\hat{y}$, $\vec{E} = -E_0\hat{x}$, $\vec{B} = B_0\hat{z}$

Q13. The circuit shown below is in a uniform magnetic field that is into the page and is decreasing in magnitude at the rate of $150\ Tesla/\sec$. The ammeter reads



(a) 0.15 A (b) 0.35 A (c) 0.50 A (d) 0.65 A

Q14. The primitive translation vectors of a hexagonal lattice may be taken as

$$\vec{a}' = \frac{a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j},\ \vec{b}' = \frac{-a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j} \text{ and } \vec{c}' = c\hat{k}$$

The volume of the reciprocal lattice is

(a) $\frac{8\pi^3}{\sqrt{3}a^2c}$ (b) $\frac{32\pi^3}{\sqrt{3}a^2c}$ (c) $\frac{16\pi^3}{\sqrt{3}a^2c}$ (d) $\frac{4\pi^3}{\sqrt{3}a^2c}$

Q15. A particle in S.H.M. has velocities u_1 and u_2 when displacements from the mean positions are x_1 and x_2 respectively. The time period of the oscillation is

(a) $2\pi\sqrt{\frac{x_2 - x_1}{u_1 - u_2}}$ (b) $2\pi\sqrt{\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}}$ (c) $2\pi\sqrt{\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}}$ (d) $2\pi\sqrt{\frac{x_1^2 - x_2^2}{u_1^2 - u_2^2}}$

Q16. A two-state quantum system has energy eigenvalues $\pm E$ corresponding to the normalized states $|\psi_{\pm}\rangle$. At time $t = 0$, the system is in quantum state $\frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$ then wave function after time $t = \frac{h}{2E}$ is

(a) $\frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$ (b) $-\frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$

(c) $\frac{1}{\sqrt{2}}\left[-|\psi_+\rangle + |\psi_-\rangle\right]$ (d) $\frac{1}{\sqrt{2}}\left[|\psi_+\rangle - |\psi_-\rangle\right]$

Q17. Consider a particle of mass m moving in one dimension under a force with potential $U(x) = k(2x^3 - 5x^2 + 4x)$ where $k > 0$. If the particle oscillates about the stable equilibrium point then Time period of oscillation is given by

(a) $2\pi\sqrt{\frac{2m}{k}}$ (b) $\pi\sqrt{\frac{2m}{k}}$ (c) $2\pi\sqrt{\frac{m}{k}}$ (d) $\pi\sqrt{\frac{m}{k}}$

Q18. A particle of mass m interact with potential $V(x) = \begin{cases} \infty, & x \le 0 \\ -V_0, & 0 \le x \le a \\ 0, & x \ge a \end{cases}$ if particle has energy $-V_0 < E < 0$ then wave vector k in the region $0 < x < a$ is given by

(a) $k = \sqrt{\frac{2mE}{\hbar^2}}$ (b) $k = \sqrt{\frac{2m(E - V_0)}{\hbar^2}}$

(c) $k = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$ (d) $k = \sqrt{\frac{2m(V_0 + E)}{\hbar^2}}$

Q19. A car tyre having air $(\gamma = 1.4)$ at $2\,atm, 300\,K$, suddenly bursts. The final temperature of the air is

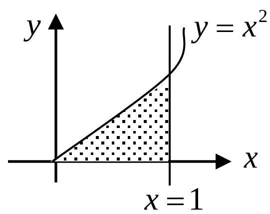
(a) $-26.9^0\,C$ (b) $26.9^0\,C$ (c) $0^0\,C$ (d) $30^0\,C$

Q20. A particle with rest mass M is at rest and decays into two particles of equal rest masses $\frac{2}{5}M$ which move along the z axis. Their velocities are given by

(a) $\vec{v}_1 = \vec{v}_2 = (0.8c)\hat{z}$ (b) $\vec{v}_1 = -\vec{v}_2 = (0.8c)\hat{z}$

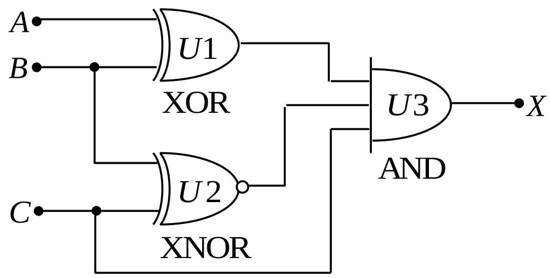
(c) $\vec{v}_1 = -\vec{v}_2 = (0.6c)\hat{z}$ (d) $\vec{v}_1 = (0.6c)\hat{z};\ \vec{v}_2 = (-0.8c)\hat{z}$

Q21. The Shaded region shown in the figure is bounded by the curve $y = x^2$, line $x = 1$ and the line $y = 0$. If the mass per unit area of the region in a certain unit is given by $\sigma(x, y) = 12xy$, then the mass of the shaded region in that same unit is



(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

Q22. For the logic circuit shown in figure, the required input condition (A, B, C) to make the output $(X)=1$ is,



(a) 1,0,1

(b) 0,0,1

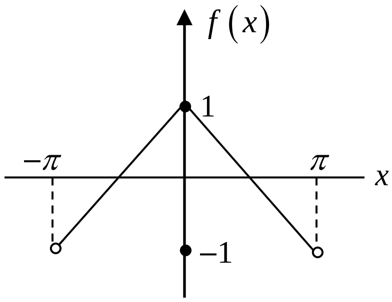
(c) 1,1,1

(d) 0,1,1

Q23. The graph of a periodic function $f(x)$ for one period is shown in the figure.

If the Fourier series of this function is written as

$$f(x)=a_0+\sum_{n=1}^{\infty} a_n \cos nx+\sum_{n=1}^{\infty} b_n \sin nx$$



then which of the following options is Incorrect. ?

(a) The value of a_0 is 0.

(b) The value of $a_3=\dfrac{8}{9\pi^2}$.

(c) The sum of series $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\ldots$ is $\dfrac{\pi^2}{8}$

(d) The coefficient of $\cos 5x$ is $\dfrac{4}{25\pi^2}$.

Q24. A projectile of mass $3m$ is projected with $20\sqrt{2}\ m/\text{sec}$ at 45^0 with ground. At highest point it explodes in 2 pieces of $2m$ and $1m$. Mass $2m$ falls at a distance of $100m$ from point of projection. The distance of second mass from point of projection where it strikes the ground is ($g=10\,m/\text{sec}^2$)

(a) $40m$ (b) $20m$ (c) $30m$ (d) $10m$

Q25. If the moment of inertia of a disc about an axis tangentially and parallel to its surface be I, then what will be the moment of inertia about the axis tangential but perpendicular to the surface:

(a) $\dfrac{6}{5}I$ (b) $\dfrac{3}{4}I$ (c) $\dfrac{3}{2}I$ (d) $\dfrac{5}{4}I$

Q26. A heat engine converts one eighth of heat supplied into work. If sink temperature is reduced by $95^0 C$, the efficiency is doubled. The source and sink temperature initially are

(a) $760\,K,\ 665\,K$ (b) $760\,K,\ 556\,K$

(c) $670\,K,\ 556\,K$ (d) $760\,K,\ 300\,K$

Q27. In the Maclaurin's series of $f(x)=\ln\left(\frac{1-x}{1+x}\right)$ the coefficient of x^7 is

(a) $-\frac{1}{3}$ (b) $\frac{1}{5}$ (c) $\frac{2}{5}$ (d) $-\frac{2}{7}$

Q28. Consider N non-interacting distinguishable particles in a two level system at temperature T. The energies of the levels are 0 and ε, when $\varepsilon>0$. In the high temperature limit $(k_B T>\varepsilon)$, what is the population of particles in the level with energy 0?

(a) N (b) $N/2$ (c) $N/4$ (d) $3N/4$

Q29. An alpha particle of energy $5\ MeV$ is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of

(a) $10^{-10}\,m$ (b) $10^{-12}\,m$ (c) $10^{-13}\,m$ (d) $10^{-14}\,m$

Q30. A plane of Miller indices (421) in orthorhombic crystal with $a:b:c=4:3:2$ makes intercept along x, y, z axis

(a) $1\overset{0}{A}, 3\overset{0}{A}, 2\overset{0}{A}$ (b) $1\overset{0}{A}, 1.5\overset{0}{A}, 4\overset{0}{A}$

(c) $2\overset{0}{A}, 3\overset{0}{A}, 4\overset{0}{A}$ (d) $2\overset{0}{A}, 1.5\overset{0}{A}, 2\overset{0}{A}$

SECTION B

Multiple Select Type Questions (MSQ)

Q31 – Q40 Carry Two Marks each (No negative marking for any wrong answer)

Q31. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current *I*. Then which of the following statements are true?

(a) Magnetic field inside the solenoid is uniform

(b) Magnetic field outside the solenoid is zero

(c) The vector potential $\vec{A}$ varies as $\frac{1}{r}$ inside the solenoid

(d) The vector potential $\vec{A}$ varies as r outside the solenoid

Q32. An oscillating voltage $V(t) = V_0 \sin \omega t$ is applied across a parallel plate capacitor having a plate separation *d*. Then which of the following statement is true?

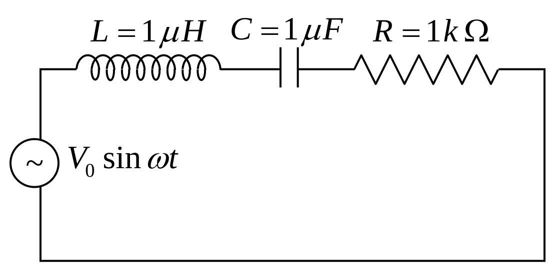
(a) Peak value of the displacement current density is $\frac{\varepsilon_0}{d} V_0 \omega$

(b) The average value of the displacement current density is $\frac{2}{\pi}\left(\frac{\varepsilon_0}{d} V_0 \omega\right)$

(c) The average value of the displacement current density is zero.

(d) The r.m.s value of the displacement current density is $\frac{\varepsilon_0}{\sqrt{2}d} V_0 \omega$

Q33. In the circuit shown $L = 1\mu H, C = 1\mu F$ and $R = 1k\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



(a) The frequency at which the current will be in the phase with the voltage is independent of R.

(b) At $\omega \approx 0$ the current flowing through the circuit becomes nearly zero.

(c) At $\omega >> 10^6\ rad\ s^{-1}$, the circuit behave like a capacitor.

(d) The current will be in phase with the voltage if $\omega = 10^4\ rad\ s^{-1}$

Q34. The potential of two dimensional Harmonic oscillator is given by $V(x,y)=\frac{1}{2}m\omega^2(x^2+y^2)$. Then which of the following is correct statement

(a) The ground state energy is given by $\frac{\hbar\omega}{2}$

(b) The first excited state is doubly degenerate

(c) If three electrons of spin $1/2$ are placed into the oscillator, then ground state configuration have energy $4\hbar\omega$

(d) If three boson of spin 1 are placed into the oscillator, then ground state configuration have energy $3\hbar\omega$

Q35. Light described by the equation $E=(90V/m)\left[\sin\left(6.28\times10^{15}s^{-1}\right)t+\sin\left(12.56\times10^{15}s^{-1}\right)t\right]$ is incident on a metal surface. The work function of the metal is $2.0\,eV$. Which of the following is correct.

(a) The threshold frequency is approximately $f=4.9\times10^{15}\sec^{-1}$

(b) The maximum kinetic energy correspond to frequency $\omega=9.42\times10^{15}\sec^{-1}$

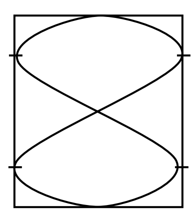
(c) The stopping potential is $8.24eV$

(d) Maximum kinetic energy is $6.24eV$

Q36. Which of the following planes are present in face central cubic (fcc) lattice?

(a) (111) (b) (200) (c) (220) (d) (310)

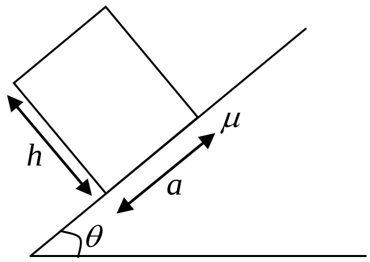
Q37. When a particle oscillates simultaneously under the action of two S.H.M. at right angle traces the curve as shown in the figure. Which of the following correctly represent the phase difference between two S.H.M.



(a) $\delta=0$ (b) $\delta=\pi$

(c) $\delta=2\pi$ (d) $\delta=3\pi$

Q38. A block with a square base measuring $a \times a$, and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will:



(a) topple before sliding if $\mu > \frac{a}{h}$ (b) never topple

(c) slide before toppling if $\mu > \frac{a}{h}$ (d) slide before toppling if $\mu < \frac{a}{h}$

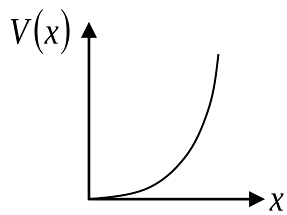
Q39. A particle is constrained to move in a truncated harmonic potential well ($x > 0$) as shown in the figure. Which one of the following statements is **NOT CORRECT**?

(a) The parity of the first excited state is even

(b) The parity of the ground state is even

(c) the ground state energy is $\frac{1}{2}\hbar\omega$

(d) The first excited state energy is $\frac{7}{2}\hbar\omega$



Q40. A jet of gas consists of molecules of mass m, speed v and number density n all moving co-linearly. This jet hits a wall at an angle θ to the normal. Then which of the following is Correct?

(a) Change in momentum is in direction of perpendicular to wall

(b) The change in momentum is proportional to $\cos\theta$

(c) The pressure exerted on the wall by the jet assuming elastic collision will be proportional to $\cos\theta$

(d) Maximum pressure is $p' = 2mnv^2$, for $\theta = 0^0$

SECTION C

Numerical Answer Type Questions (NAT)

Q41 – Q50 Carry One Mark each (No negative marking for any wrong answer).

Q41. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of $1m$ from the diode is............V/m

Q42. Two large nonconducting sheets one with a fixed uniform positive charge and another with a fixed uniform negative charge are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities are $\sigma_+ = 3.4\mu\ C/m^2$ for the positively charged sheet and $\sigma_- = 2.1\mu\ C/m^2$ for the negatively charged sheet. The electric field in the region between the sheets is .... $\times 10^4\ N/C$

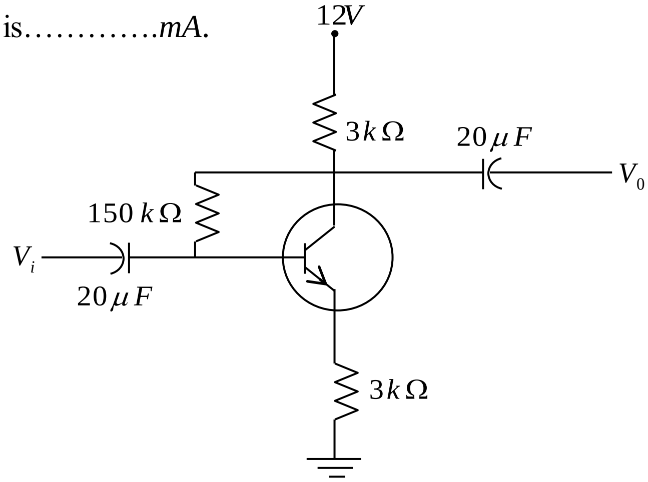
Q43. Consider the quantum state $|\Psi\rangle = |\psi_1\rangle + 2|\psi_2\rangle + 3|\psi_3\rangle$, where $|\psi_1\rangle, |\psi_2\rangle$ and $|\psi_3\rangle$ are normalized eigenstates of a hermitian operator H with eigenvalues 5, 2 and 4 respectively. The expectation value of H in the state $|\Psi\rangle$ is.........

Q44. A radioactive sample emits n β-particles in $2\sec$. In next $2\sec$ it emits $0.75n$ β-particles, then the mean life of the sample is.......seconds $(\ln 2 = 0.693, \ln 3 = 1.0986)$

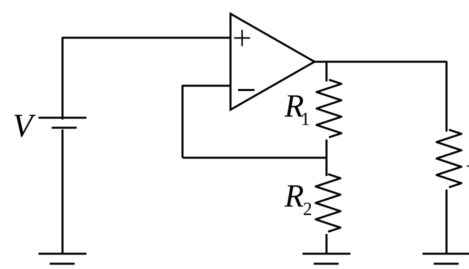
Q45. The de Broglie wavelength of a relativistic electron having $1\,MeV$ of energy is............. $\times 10^{-12}\ m$. (Take the rest mass energy of the electron to be $0.5\ MeV$. Plank constant $= 6.63\times 10^{-34}\ Js$, speed of light $= 3\times 10^8\ m/s$, Electronic charge $= 1.6\times 10^{-19}\ C$)

Q46. The ratio of the atomic density of plane $(1\,1\,1)$ to plane $(1\,0\,0)$ in BCC crystal is ....

Q47. The current gain of the transistor in the following circuit is $\beta_{dc} = 100$. The value of collector current I_C is…………mA.



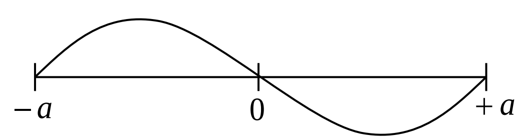
Q48. Consider an ideal operational amplifier as shown in the figure below with $R_1 = 5k\Omega, R_2 = 1k\Omega, R_L = 100k\Omega$. For an applied input voltage $V = 20\,mV$, the current passing through R_2 is………….. μA.



Q49. In the ideal YDSE, when a glass plate $(\mu = 1.5)$ of thickness $'t'$ is introduced in the path of one of the interfering beams $\left(\lambda = 6000A^0\right)$ the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass plate is………… A^0.

Q50. $\psi(x)$ is the eigenfunction of a particle moving freely back and forth along the x - axis between impenetrable walls located at $x = -a$ and $x = a$ as shown in the figure . The potential energy is given by

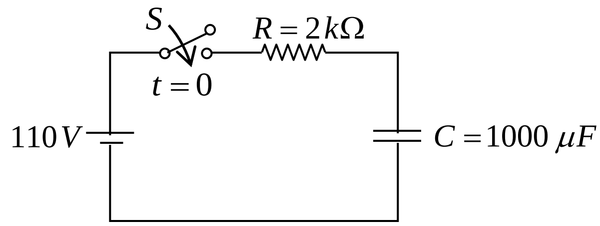
$$V(x) = \begin{cases} 0, & |x| < a \\ \infty, & \text{otherwise} \end{cases}$$



The energy of the particle is $2eV$ when it is in the quantum state associated with this eigenfunction. When three electrons are put in the well, then the lowest possible energy is ……… eV.

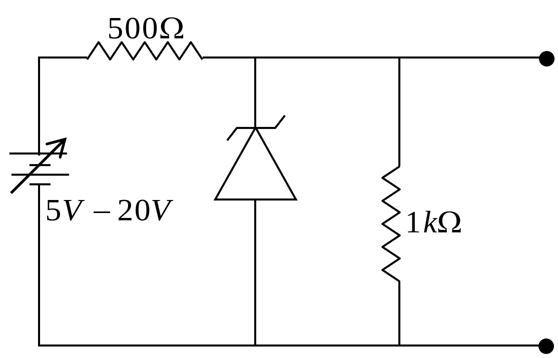
Q51 – Q60 Carry Two Marks each (No negative marking for any wrong answer).

Q51. The RC series circuit is shown in figure below. The capacitor is initially uncharged. At time $t = 0$, the switch S is closed. The voltage across capacitor $1\sec$ after the circuit is closed is…………...Volts



Q52. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is…………degree.

Q53. A variable power supply ($5V - 20V$) is connected to a Zener diode specified by a breakdown voltage of $10V$ (see figure). Then the maximum power dissipated across the zener diode is…………. mA.



Q54. A satellite of mass m is orbiting in a circular orbit of radius r and velocity v_0 around the earth of mass M. Due to internal explosion, the satellite breaks into two fragments each of mass $m/2$.In frame of reference of satellite, the two fragments appears to move radially along the line joining the original satellite and center of the each satellite has a velocity $\frac{v_0}{2}$. If the total energy of each fragment is $-\alpha\frac{GMm}{r}$, then the value of α is__________ (Answer must be upto two decimal points)

Q55. An ensemble of systems is in thermal equilibrium with a reservoir for which $kT = 0.025\,eV$. State A has an energy that is $0.1\,eV$ above that of state B. If it is assumed the systems obey Maxwell - Boltzmann statistics and that the degeneracies of the two states are the same, then the ratio of the number of systems in state A to the number in state B is $e^{-\alpha}$. Then the value of α is ______________

Q56. Three sinusoidal waves have the same amplitude 1 unit, $\frac{1}{2}$ unit and $\frac{1}{3}$ unit while their phase angles are $0, \frac{\pi}{2}$ and π respectively. the amplitude of resultant wave is.........unit.

Q57. The moment of inertia of a disc about one of its diameters is I_M. The mass per unit area of the disc is proportional to the distance from its centre. If the radius of the disc is R and its mass is M, the value of I_M is αMR^2 then value of α is ....

Q58. A ball of mass m moves at speed v and makes head on collision with identical ball at rest. Kinetic energy of the balls after collision is $\frac{3}{4}$ of original kinetic energy. The value of e is ....(Upto two digits after decimal)

Q59. An iron cube of $60\,mm$ edge is in equilibrium in two liquids mercury and water. The length of edge in water is .... mm.

(Use density, $\rho_{Hg} = 13.6\,g/cm^3, \rho_{Fe} = 7.874\,g/cm^3$ and $\rho_{\text{water}} = 1\,gm/cm^3$)

Q60. The magnitude of change in melting point of ice when it is subjected to a pressure of $100\,atm$. is .... ^0C (ice density $0.92\,gm/cm^3$, $L_f = 80\,cal/gm$)