(Solution)

IIT – JAM FULL LENGTH TEST – 01

20-01-2018

Ans. 1: (c)

Solution: $\sigma_b = \sigma_d \Rightarrow \frac{Q}{4\pi b^2} = \frac{2Q}{4\pi d^2} \Rightarrow d = \sqrt{2}b$

Ans. 2: (c)

Ans. 3: (d)

Solution: Here $\phi = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$.

Unit normal vector is $\frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}$.

So, $\vec{\nabla}\phi = \left(i\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1\right) = \frac{2x\hat{i}}{a^2} + \frac{2y\hat{j}}{b^2} + \frac{2z\hat{k}}{c^2}$

$$\vec{\nabla}\phi\Big|_{\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)} = \frac{2}{a^2\sqrt{3}}\hat{i} + \frac{2}{b^2\sqrt{3}}\hat{j} + \frac{2}{c^2\sqrt{3}}\hat{k}$$

$$|\vec{\nabla}\phi| = \sqrt{\frac{4}{3a^4} + \frac{4}{3b^4} + \frac{4}{3c^4}} = \frac{2}{\sqrt{3}}\sqrt{\frac{b^4c^4 + a^4c^4 + a^4b^4}{a^4b^4c^4}}$$

$$\frac{\vec{\nabla}\phi}{|\vec{\nabla}\phi|}\Bigg|_{\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)} = \frac{\frac{2}{a^2\sqrt{3}}\hat{i} + \frac{2}{b^2\sqrt{3}}\hat{j} + \frac{2}{c^2\sqrt{3}}\hat{k}}{\frac{2}{\sqrt{3}}\sqrt{\frac{b^4c^4 + a^4c^4 + a^4b^4}{a^4b^4c^4}}} = \frac{b^2c^2\hat{i} + c^2a^2\hat{j} + a^2b^2\hat{k}}{\sqrt{b^4c^4 + a^4c^4 + a^4b^4}}$$

Ans. 4: (c)

Solution: $v = \sqrt{\frac{T}{\mu}}$

If v_1 and v_2 are the speeds corresponding to tension in the string T_1 and T_2 respectively, then

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow v_2 = v_1\sqrt{\frac{T_2}{T_1}}$$

$$\therefore v_2 = 500\times\sqrt{\frac{78.4}{19.6}} = 500\times 2 = 1000\, m/s$$

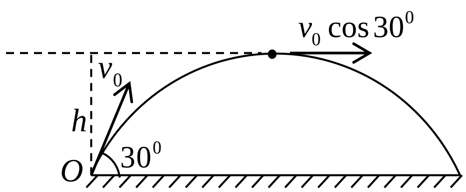
Ans. 5: (a)

Solution: $Q = \omega\tau \Rightarrow \tau = \dfrac{Q}{\omega} = \dfrac{8\times10^4}{2\times3.14\times512} = 24.88\sec$

Ans. 6: (a)

Solution: $L_0 = mvr_\perp$

At the highest point,



$$v = \text{speed } = v_0\cos 30^0 = \frac{\sqrt{3}v_0}{2}$$

$$r_\perp = h = \frac{v_0^2\sin^2 30^0}{2g} = \frac{v_0^2}{8g}$$

$$\Rightarrow \quad L_0 = m\left(\frac{\sqrt{3}v_0}{2}\right)\left(\frac{v_0^2}{8g}\right) = \frac{\sqrt{3}mv_0^3}{16g}$$

Ans. 7: (b)

Solution: Stopping potential is the negative potential which stops the emission of $(K.E)_{\max}$ electrons when applied.

$\therefore$ Stopping potential = 4 volt

Ans. 8: (d)

Solution: $\beta = \dfrac{DA}{d}$ and $\beta' = \dfrac{(2D)\lambda}{d/2} = 4\beta$

Ans. 9: (d)

Solution: we have $\dfrac{dy}{dx} + (\tanh x)y = 2e^x$ This is linear differential equation of the first order.

Integrating factor $= e^{\int \tanh x dx} = e^{\ln\cosh x} = \cosh x$

Hence the solution of this equation is $y.\cosh x = \int (2e^x)\cosh x dx + c$

$$\Rightarrow y.\frac{e^x + e^{-x}}{2} = 2\int \frac{e^x\left(e^x + e^{-x}\right)}{2}dx + c_0$$

$$y.\frac{e^{x}+e^{-x}}{2}=\int\left(e^{2x}+1\right)dx+c_0$$

$$\Rightarrow y.\frac{e^{x}+e^{-x}}{2}=\frac{e^{2x}}{2}+x+c_0$$

$$\Rightarrow y=\frac{\left(e^{2x}+2x+c\right)}{e^{x}+e^{-x}}, \text{ where } 2c_0=c$$

Ans. 10: (a)

Solution: $\cos\left[\log\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^{i}\right]=\cos\left[\log\left(e^{i\pi/2}\right)^{i}\right]=\cos\left[\log e^{-\pi/2}\right]$

$$=\cos\left\{\left(-\pi/2\right).\log_e^e\right\}=\cos\left(\frac{\pi}{2}\right)=0 \quad \Rightarrow z^2=0$$

Ans. 11: (c)

Solution: $B_A=\frac{\mu_0}{2}\frac{i}{\left(l/2\pi\right)}$ and $B_B=\left[\frac{\mu_0}{4\pi}\frac{i}{l/8}\left(\sin 45^\circ+\sin 45^\circ\right)\right]\times 4$

$$\frac{B_A}{B_B}=\frac{\pi^2}{8\sqrt{2}}$$

Ans. 12: (d)

Solution: For constant velocity $q\left[\vec{E}+\left(\vec{V}\times\vec{B}\right)\right]=0$

Ans. 13: (b)

Solution: From Ohm's Law $V-\varepsilon=IR$, one can obtain the current. (Note that V = 5.0 V is the voltage of the battery. The voltage induced acts to oppose this emf from the battery.)

The problem gives $\frac{dB}{dt}=150T/s$. The area is just $0.01\ m^2$.

Thus, the induced emf is, $\varepsilon=\frac{dB}{dt}\ A=150\times 0.01=1.5$

Thus, $V-\varepsilon=3.5=IR\Rightarrow I=0.35A$, since $R=10\Omega$.

Ans. 14: (c)

Solution: Volume of the primitive cell is

$$V' = \vec{a}' \cdot \left(\vec{b}' \times \vec{c}'\right)$$

$$= \left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j}\right) \cdot \left(\frac{-a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j}\right) \times c\hat{k}$$

$$= \left(\frac{a}{2}\hat{i} + \frac{\sqrt{3}a}{2}\hat{j}\right) \cdot \left(\frac{ac}{2}\hat{j} + \frac{\sqrt{3}ac}{2}\hat{i}\right) = \frac{\sqrt{3}}{2}a^2c$$

Volume of the reciprocal cell is

$$V^* = \frac{(2\pi)^3}{V'} = \frac{(2\pi)^3}{\frac{\sqrt{3}}{2}a^2c} = \frac{16\pi^3}{\sqrt{3}a^2c}$$

Ans. 15: (b)

Solution: $u_1 = \omega\sqrt{A^2 - x_1^2}$, $u_2 = \omega\sqrt{A^2 - x_2^2}$

$$\Rightarrow \frac{u_1^2}{\omega^2} = A^2 - x_1^2 \text{ and } \frac{u_2^2}{\omega^2} = A^2 - x_2^2$$

$$\Rightarrow \frac{u_1^2}{\omega^2} = \frac{u_2^2}{\omega^2} + x_2^2 - x_1^2 \Rightarrow \frac{u_1^2 - u_2^2}{\omega^2} = x_2^2 - x_1^2$$

$$\Rightarrow \omega = \sqrt{\frac{u_1^2 - u_2^2}{x_2^2 - x_1^2}}$$

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{u_1^2 - u_2^2}}$$

Ans. 16: (b)

Solution: $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$

And $|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{-\frac{iEt}{\hbar}} + |\psi_-\rangle e^{\frac{iEt}{\hbar}}\right]$

At $t = \frac{h}{2E}$, $|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|\psi_+\rangle e^{-i\pi} + |\psi_-\rangle e^{i\pi}\right] = -\frac{1}{\sqrt{2}}\left[|\psi_+\rangle + |\psi_-\rangle\right]$

Ans. 17: (b)

Solution: $\frac{dU}{dx}=0\Rightarrow k\left(6x^2-10x+4\right)=0\Rightarrow x=1, x=\frac{2}{3}$

$$\frac{d^2U}{dx^2}=k\left(12x-10\right)$$

For $x=1$, $\frac{d^2U}{dx^2}=2k>0$ and for $x=\frac{2}{3}$, $\frac{d^2U}{dx^2}=-4k<0$ So

$$\omega=\sqrt{\frac{2k}{m}}\Rightarrow\frac{2\pi}{T}=\sqrt{\frac{2k}{m}}\Rightarrow T=2\pi\sqrt{\frac{m}{2k}}\Rightarrow T=\pi\sqrt{\frac{2m}{k}}$$

Ans. 18: (d)

Solution: $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}-V_0\psi=E\psi\Rightarrow\frac{d^2\psi}{dx^2}+\frac{2m\left(E+V_0\right)}{\hbar^2}\psi=0$

$$k=\sqrt{\frac{2m\left(V_0+E\right)}{\hbar^2}}$$

Ans. 19: (a)

Solution: As process is adiabatic, $T^{\gamma}/P^{\gamma-1}=C$

$$\Rightarrow\frac{T_1^{\gamma}}{P_1^{\gamma-1}}=\frac{T_2^{\gamma}}{P_2^{\gamma-1}}$$

$$T_2=300\times\left(\frac{1}{2}\right)^{\left(\frac{1.4-1}{1-4}\right)}=246.1K$$

$$T_2=-26.9^0C$$

Ans. 20: (c)

Solution: $M\rightarrow\frac{2}{5}M+\frac{2}{5}M$

From momentum conservation

$$0=\vec{P}_1+\vec{P}_2\Rightarrow\vec{P}_1=-\vec{P}_2\Rightarrow\left|P_1\right|=\left|P_2\right|$$

From energy conservation

$$E=E_1+E_2$$

$$\Rightarrow Mc^2 = \frac{2}{5}\frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{2}{5}\frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Rightarrow Mc^2 = \frac{4}{5}\frac{Mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\left(1-\frac{v^2}{c^2}\right) = \frac{16}{25} \Rightarrow \frac{v^2}{c^2} = \frac{9}{25} \Rightarrow v = \frac{3}{5}c = 0.6c$$

Ans. 21: (a)

Solution: By definition $m = \iint_R \sigma(x,y)dA$

$$\Rightarrow m = 12\int_0^1\int_0^{x^2} xydydx = 12\int_0^1 \left[\frac{xy^2}{2}\right]_0^{x^2} dx = 12\int_0^1\left(\frac{x^5}{2}-0\right)dx = 12\left[\frac{x^6}{2.6}\right]_0^1 = 12\left[\frac{x^6}{12}\right]_0^1 = 1$$

Ans. 22: (d)

Solution: XOR is inequality comparator and XNOR is equality comparator. In AND gate output will be high when all the input is 1.

Ans. 23: (d)

Solution: The given function can be written as $f(x) = \begin{cases} 1+\frac{2x}{\pi}, -\pi \le x \le 0 \\ 1-\frac{2x}{\pi}, 0 < x < \pi \end{cases}$

$f(x)$ can be written as $f(x) = 1-\frac{2}{\pi}|x|, \quad -\pi \le x \le \pi$

For constant function (1) we have $a_0 = 1,\ a_n = 0,\ b_n = 0$

For $|x|, -\pi \le x \le \pi$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} xdx = \frac{1}{\pi}\cdot\frac{\pi^2}{2} = \frac{\pi}{2}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos nxdx = \frac{2}{\pi}\left[\frac{x\sin nx}{n} + \frac{1}{n^2}\cos nx\right]_0^{\pi} \Rightarrow a_n = \frac{2}{\pi}\left[\frac{1}{n^2}(\cos n\pi - 1)\right]$$

For even n, $a_n = 0$

For odd n, $\quad a_n = \frac{2}{\pi}\cdot\frac{1}{n^2}(-2) = -\frac{4}{\pi n^2}$

Thus for the function $f(x) = 1 - \frac{2}{\pi}|x|$

We have

$$a_0 = 1 - \frac{2}{\pi}\left(\frac{\pi}{2}\right) = 0$$

$$a_n = 0 + 0 = \text{ for even } n$$

$$a_n = 0 - \frac{2}{\pi}\left(-\frac{4}{\pi n^2}\right) = \frac{8}{\pi^2 n^2} \text{ for odd } n.$$

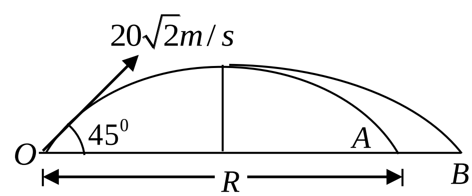
Hence the fourier series of $f(x)$ is $f(x) = \frac{8}{\pi^2}\sum_{n=1,3,5}^{\infty}\frac{1}{n^2}\cos nx$

Putting $x = 0$ in this series we obtain

$$1 = \frac{8}{\pi^2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + ...\right) \Rightarrow 1 + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$$

Ans. 24: (a)

Solution:



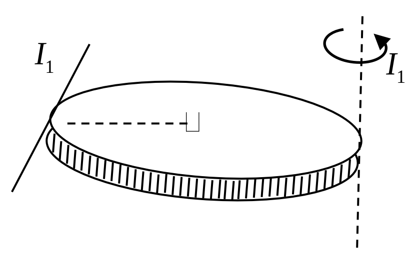
$$R = \frac{v^2 \sin 2\theta}{g} = \frac{20\sqrt{2} \times 20\sqrt{2} \times 1}{10} = 80\,m$$

Centre of mass will strike the ground at A

$$x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$\Rightarrow 80 = \frac{m x_1 + 2m \times 100}{m + 2m} = 40\,m \Rightarrow x_1 = 40m$$

Ans. 25: (a)



Solution: $I_1 = \left(\frac{1}{4}mr^2\right) + mr^2 = \frac{5mr^2}{4} = I$

$$\Rightarrow \qquad I_2 = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2 = \frac{6I}{5}$$

Ans. 26: (a)

Solution: $\eta_1 = \frac{W}{Q_1} = \frac{1}{8} = 1 - \frac{T_2}{T_1}$, $\eta_2 = \frac{1}{4} = 1 - \frac{T_2 - 95}{T_1}$

On solving $T_1 = 760\,K$ and $T_2 = 665\,K$

Ans. 27: (d)

Solution: $f(x) = \ln(1-x) - \ln(1+x) = \ln[1+(-x)] - \ln[1+x]$

$$= \left[(-x) - \frac{(-x)^2}{2} + \frac{(-x)^3}{3} - \frac{(-x)^4}{4} + \frac{(-x)^5}{5} + ...\right]$$

$$-\left[x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + ...\right]$$

$$= -2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \cdots\right)$$

Ans. 28: (b)

Solution: $P(0) = \frac{1}{1+e^{-\varepsilon/k_BT}}$

Population with energy 0

$$= N \cdot \frac{1}{1+e^{-\varepsilon/k_BT}}$$

$\lim \varepsilon/k_BT \ll 1$

$$= \frac{N}{2}$$

Ans. 29: (d)

Solution: Energy is conserved.

Loss in kinetic energy = Gain in potential energy

$$\frac{1}{4\pi\varepsilon_0}\frac{(Ze)(2e)}{r_{\min}} = 5\times\left(1.6\times10^{-13}\right)J \Rightarrow r_{\min} = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{5\times1.6\times10^{-13}}$$

or $r_{\min} = \frac{\left(9\times10^9\right)(2)(92)\left(1.6\times10^{-19}\right)^2}{5\times1.6\times10^{-13}} \Rightarrow r_{\min} = 5.3\times10^{-14}m \Rightarrow r_{\min} = 5.3\times10^{-12}cm$

The distance of closest approach is of the order of $10^{-12}cm$ or $10^{-14}m$

Ans. 30: (c)

Solution: $p:q:r=\frac{a}{h}:\frac{b}{k}:\frac{c}{l}=\frac{4}{4}:\frac{3}{2}:\frac{2}{1}=1:\frac{3}{2}:2$

or $p:q:r=2:3:4$

Thus, possible value of intercepts are

$$p=2\overset{0}{\mathrm{A}},\quad q=3\overset{0}{\mathrm{A}},\quad r=4\overset{0}{\mathrm{A}}$$

Ans. 31: (a), (b)

Ans. 32: (a), (c) and (d)

Solution: Displacement current density $J_d=\varepsilon_0\frac{\partial E}{\partial t}=\frac{\varepsilon_0}{d}\frac{\partial V(t)}{\partial t}=\frac{\varepsilon_0\omega V_0\cos\omega t}{d}$

Ans. 33: (a), (b)

Solution: $I=\frac{V_0}{\sqrt{\left(L\omega-\frac{1}{C\omega}\right)+R^2}}$

$f_R=\frac{1}{\sqrt{LC}}=10^6\ rad\ s^{-1}\Rightarrow$ Resonance frequency

If ω □ $10^6\ rad/s$, then circuit behaves as inductive circuit.

Ans. 34: (b), (c) and (d)

Solution: $E_n=(n+1)\hbar\omega$ and degeneracy of nth state is $n+1$

Two electrons will be $n=0$ and one electron in $n=1$

$E=2\times\hbar\omega+1\times2\hbar\omega=4\hbar\omega$

Boson will not follow Pauli exclusion principle so all three boson will be in ground state so energy $3\hbar\omega$

Ans. 35: (a) , (d)

Solution: For threshold frequency $hf=2\times1.6\times10^{-19}\Rightarrow f=\frac{2\times1.6\times10^{-19}}{6.6\times10^{-34}}=4.9\times10^{15}\sec^{-1}$

$K_{\max}=\hbar\omega-W$

For given wave maximum kinetic energy is for highest ω so $\omega=12.56\times10^{15}\sec^{-1}$

$$\hbar\omega=\frac{6.6\times10^{-34}J\,s\times12.56\times10^{15}s^{-1}}{2\pi}=\frac{82.8\times10^{-19}J}{6.28\times1.6\times10^{-19}}eV=8.24eV$$

$K_{\max} = \hbar\omega - W \Rightarrow 8.24eV - 2eV = 6.24eV$

Ans. 36: (a), (b), (c)

Solution: In FCC lattice only unmixed $(h\,k\,l)$ are present. $(111),(200)$ and (220) are unmixed $(h\,k\,l)$, therefore they are present while (310) is mixed $(h\,k\,l)$ and it is absent.

Ans. 37: (a), (b), (c), (d)

Solution: For $x = a\sin(2\omega t + \delta)$ and $y = b\sin(\omega t)$

$$= 2a\sin\omega t\cos\omega t$$

$$\therefore \frac{x}{a} = 2\sin\omega t\cos\omega t = \frac{2y}{b}\sqrt{1-\frac{y^2}{b^2}}$$

$$\Rightarrow \frac{x^2}{a^2} = \frac{4y^2}{b^2}\left(1-\frac{y^2}{b^2}\right) = \frac{-4y^2}{b^2}\left(\frac{y^2}{b^2}-1\right)$$

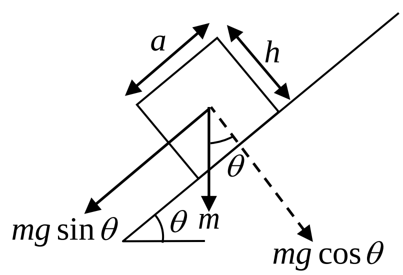
$$\frac{x^2}{a^2} + \frac{4y^2}{b^2}\left(\frac{y^2}{b^2}-1\right) = 0$$

This equation represent the figure 8.

Ans. 38: (a), (d)

Solution: For no slipping,

$$mg\sin\theta < \mu mg\cos\theta \;\Rightarrow\; \tan\theta < \mu$$



For no toppling,

$$mg\sin\theta\frac{h}{2} < mg\cos\theta\frac{a}{2}$$

$$\Rightarrow \qquad \tan\theta < \frac{a}{h}$$

Combine to get: $\mu > \frac{a}{h}$ for toppling before slipping.

Ans. 39: (a), (b), (c)

Solution: There is only odd parity. Ground state is $\frac{3}{2}\hbar\omega$ and first excited $= \frac{7}{2}\hbar\omega$

Ans. 40: (a), (b) and (d)

Solution: Change in momentum along $y-$direction will be cancelled out

$\because$ change in momentum along $x-$direction

$\Delta p = 2mv\cos\theta$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\frac{\Delta p}{\Delta t}}{A} = \frac{\Delta p}{A.\Delta t} = \frac{\Delta p}{A\cdot\frac{L}{v\cos\theta}} = \frac{\Delta p v\cos\theta}{A.L}$$

$$\text{Pressure } p' = \frac{2mv\cos\theta\cdot v\cos\theta N}{v}, \quad \because\left(n=\frac{N}{V}\right), \left(V = \text{Area}\times L = A\times L\right),$$

$$p' = 2mnv^2\cos^2\theta$$

Ans. 41: 2.45

Solution: Intensity, $I = \frac{1}{2}\varepsilon_0 E_0^2 c$, where E_0 is amplitude of the electric field of the light.

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$E_0 = \sqrt{\frac{2P}{4\pi r^2 c\varepsilon_0}} = 2.45V/m$$

Ans. 42: 31

Solution: Electric field between the sheet is $E = \frac{\sigma_+}{2\in_0} + \frac{\sigma_-}{2\in_0} = \frac{3.4\times10^{-6}}{2\in_0} + \frac{2.1\times10^{-6}}{2\in_0}$

$$\Rightarrow E = \frac{5.5\times10^{-6}}{2\times8.86\times10^{-12}} = 0.31\times10^6 = 31\times10^4 \ N/C$$

Ans. 43: 3.5

Solution: $\langle H\rangle = \frac{5\times1}{1+4+9} + \frac{2\times4}{1+4+9} + \frac{4\times9}{1+4+9} = \frac{5}{14} + \frac{8}{14} + \frac{36}{14} = \frac{49}{14} = 3.5$

Ans. 44: 7

Solution: Let N_0 be the number of initial number of nuclei. Then

$$n = N_0 - N_0 e^{-2\lambda} = N_0(1-e^{-2\lambda})$$

$$0.75n = N_0 e^{-2\lambda} - N_0 e^{-2\lambda} e^{-2\lambda} = N_0 e^{-2\lambda}(1-e^{-2\lambda})$$

$$\frac{0.75n}{n} = \frac{N_o e^{-2\lambda}\left(1-e^{-2\lambda}\right)}{N_o\left(1-e^{-2\lambda}\right)} \Rightarrow e^{-2\lambda} = \frac{3}{4} \Rightarrow 2\lambda = 2\ln 2 - \ln 3 \Rightarrow \lambda = 0.1438\sec^{-1}$$

$$\Rightarrow \bar{T} = \frac{1}{\lambda} = 7\sec$$

Ans. 45: 1.43

Solution: $E^2 = p^2c^2 + \left(m_0c^2\right)^2 \Rightarrow p = \sqrt{\dfrac{E^2 - \left(m_0c^2\right)^2}{c^2}} = \dfrac{\sqrt{1-0.25}}{c} = \dfrac{\sqrt{0.75}MeV}{c}$

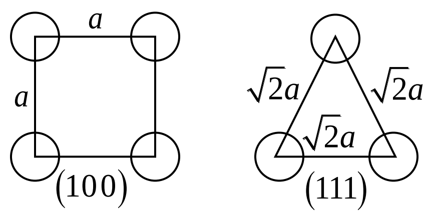
$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}\times3\times10^{8}}{\sqrt{0.75}\times1.6\times10^{-13}} = \frac{19.8\times10^{-13}}{1.38} = 14.34\times10^{-13}m = 1.43\times10^{-12}m$$

Ans. 46: 0.577

Solution: $\rho_{100} = \dfrac{n_{eff}}{a^2} = \dfrac{1}{a^2}$

$$\rho_{111} = \frac{n_{eff}}{area} = \frac{1/2}{\frac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2} = \frac{1}{\sqrt{3}a^2}$$

$$\frac{\rho_{111}}{\rho_{100}} = \frac{1/\sqrt{3}a^2}{1/a^2} = \frac{1}{\sqrt{3}} = 0.577$$



Ans. 47: 1.6

Solution: $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + \beta\left(R_C + R_E\right)} = \dfrac{12-0}{150+100(3+3)} = 0.016\,mA \Rightarrow I_C = \beta I_B = 1.6\,mA$

Ans. 48: 20

Solution: $I_2 = \dfrac{V}{R_2} = \dfrac{20}{1} = 20\mu A$

Ans. 49: 12000

Solution: $(\mu-1) = \lambda$

$$t = \frac{6000}{1.5-1} = 12000A^0$$

Ans. 50: 3

Solution: The particle is in an infinite well of length $2a$. (It's stuck forever bouncing around between the two walls.)

The number of nodes in the wave function determines the energy level. In this case, there is one load, thus this is E_2. The lowest state would be E_1.

$E_n = kn^2 eV$ for particle in a box. Given that $E_2 = 2eV$ one determines $k = 1/2eV$. For one particle system. Thus, $E_1 = 0.5eV$ for three electron system $E = 2\times 0.5eV + 1\times 2eV = 3eV$

Ans. 51: 43.2

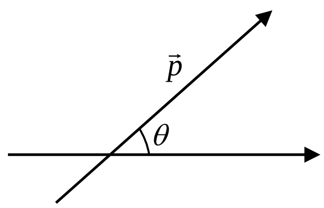
Solution: $v_C = V\left[1-\exp\left(-t/RC\right)\right] = 110\left[1-e^{\left(-\frac{1}{2000\times10^{-3}}\right)}\right]$

$= 110\left[1-e^{(-0.5)}\right] = 110(1-0.607) = 43.2\,V$

Ans. 52: 60

Solution: So from $\vec{\tau} = \vec{p}\times\vec{E}$

$\tau\hat{k} = \left(p_x\hat{i} + p_y\hat{j}\right)\times\left(E\hat{i}\right) = -p_y E\hat{k}$



$-\tau\hat{k} = \left(p_x\hat{i} + p_y\hat{j}\right)\times\left(\sqrt{3}E\hat{j}\right) = p_x\times\sqrt{3}E\hat{k}$

Thus $-\left(-p_y E\hat{k}\right) = p_x\times\sqrt{3}E\hat{k}$

$\Rightarrow \frac{p_y}{p_x} = \sqrt{3} \Rightarrow \tan\theta = \sqrt{3} \qquad \Rightarrow \theta = 60^0$

Ans. 53: 100

Solution: When $V = 5V$ $\Rightarrow$open circuit voltage $V_i = \frac{1000}{1500}\times 5 = 3.33 < V_Z = 10V \Rightarrow I_Z = 0$

$\Rightarrow P_{z,\min} = V_Z I_Z = 0$.

When $V = 20V$ $\Rightarrow$open circuit voltage $V_i = \frac{1000}{1500}\times 20 = 13.33 > V_Z = 10V$

$\Rightarrow I_Z = I_R - I_L = \frac{10}{0.5} - \frac{10}{1} = 20 - 10 = 10mA$

$\Rightarrow P_{z,\max} = V_Z I_Z = 10\times 10 = 100mW$

Ans. 54: 0.19

Solution: $\frac{mv_0^2}{r} = \frac{GMm}{r^2} \Rightarrow mv_0^2 = \frac{GMm}{r}$

Resultant velocity at the each fragment from earth $v = \sqrt{v_0^2 + \frac{v_0^2}{4}} = \frac{v_0\sqrt{5}}{2}$

So kinetic energy $T=\frac{1}{2}\frac{m}{2}v^2=\frac{5}{16}mv_0^2=\frac{5GmM}{16r}$

Potential energy is $-\frac{GmM}{2r}$

Total energy $E=T+V=-\frac{3GMm}{16r}=-0.1875\frac{GMm}{r}$

Ans. 55: 4

Solution: $P(A)\propto e^{-\beta(E+0.1eV)}$

$P(B)\propto e^{-\beta(E)}$

$P(A)/P(B)=e^{-0.1eV/(0.025eV)}=e^{-4}$

Ans. 56: 0.83

Solution: $y=\sin\omega t+\frac{1}{2}\cos\omega t-\frac{1}{3}\sin\omega t$

$=\frac{2}{3}\sin\omega t+\frac{1}{2}\cos\omega t$

$R^2=\left(\frac{2}{3}\right)^2+\left(\frac{1}{2}\right)^2=\frac{25}{36}\Rightarrow R=\frac{5}{6}$ Units

Ans. 57: 0.3

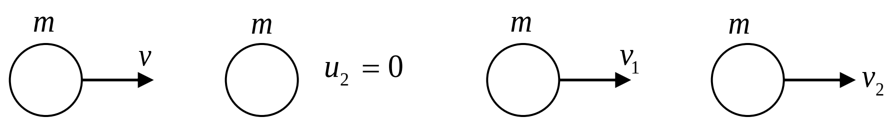
Solution: Mass density is $\sigma=cr$, $M=\int_0^r cr.rdrd\theta\Rightarrow c=\frac{3M}{2\pi R^3}$

$I_z=\int_0^R r^2dm\Rightarrow\int_0^R r^2cr.rdrd\theta=\frac{c2\pi R^5}{5}$ put value of c $I_z=\frac{3}{5}MR^2$

From perpendicular axis theorem $I_z=I_x+I_y$ and $I_x=I_y$ so $I_x=I_y=I_M=\frac{3}{10}MR^2$

Ans. 58: 0.71

Solution:



Conservation of momentum

$$mv + 0 = mv_1 + mv_2 \Rightarrow v_1 + v_2 = v$$

$$k_f = \frac{3}{4}k_i$$

$$\Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{4}\times\frac{1}{2}mv^2 + 0$$

$$\Rightarrow 4\left(v_1^2 + v_2^2\right) = 3\left(v_1 + v_2\right)^2$$

$$v_1^2 + v_2^2 - 6v_1v_2 = 0$$

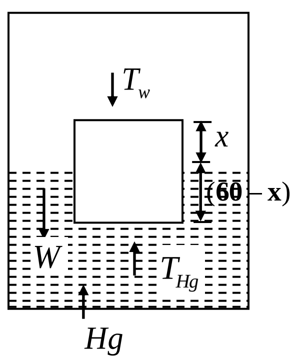
$$\Rightarrow \left(\frac{v_2}{v_1}\right)^2 - \frac{6v_2}{v_1} + 1 = 0 \Rightarrow \left(\frac{v_2}{v_1}\right) = \frac{6\pm 4\sqrt{2}}{2} = 3\pm 2\sqrt{2}$$

$$e = \frac{v_2 - v_1}{v - 0} = \frac{v_2 - v_1}{v_2 + v_1} \Rightarrow e = \frac{v_2/v_1 - 1}{v_2/v_1 + 1} = \frac{3\pm 2\sqrt{2} - 1}{3\pm 2\sqrt{2} + 1}$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

Ans. 59: $23.53\,mm$

Solution: In equilibrium



$$T_w + W = T_{Hg}$$

Where $T_w = \rho_w \; x(60)^2 g$

$$T_{Hg} = \rho_{Hg}(60 - x)(60^2)g \text{ and } W = \rho_{Fe}(60)^3 g$$

$$\therefore \rho_w x(60)^2 g + \rho_{Fe}(60)^3 g = \rho_{Hg}(60 - x)(60)^2 g$$

$$\Rightarrow \rho_w \; x + 60 \; \rho_{Fe} = (60 - x)\rho_{Hg}$$

$$\Rightarrow x = (60\ mm)\frac{\rho_{Hg} - \rho_{Fe}}{\rho_{Hg} + \rho_w}$$

$$= (60\,mm)\frac{13600 - 7874}{13600 + 1000}$$

$$= (60\,mm)\times 0.39$$

$$= 23.53\,mm$$

Ans. 60: 0.73

Solution: $\frac{dp}{dT}=\frac{L}{T(V_2-V_1)}\Rightarrow dT=\frac{Tdp(V_2-V_1)}{L}$

$T=273+0=273K$, $L=80\,cal/gm$

V_1 (Specific volume of ice at 0^0C) $=\frac{1}{0.92}gm$

V_2 (Specific volume of water at 0^0C) $=1cm^3/gm$

$$\Rightarrow dT=\frac{273\times(100-1)\times10^5\times1.018\times\left(1-\frac{1}{0.092}\right)\times10^{-3}}{80\times4187}=-0.733^0C$$