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2. ELECTROSTATIC POTENTIAL AND CAPACITANCE

PGT Physics-Practice Set

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SUMMARY

1. Electrostatic force is a conservative force. Work done by an external force (equal and opposite to the electrostatic force) in bringing a charge q from a point R to a point P is $U_p - U_R$, which is the difference in potential energy of charge q between the final and initial points.

2. Potential at a point is the work done per unit charge (by an external agency) in bringing a charge from infinity to that point. Potential at a point is arbitrary to within an additive constant, since it is the potential difference between two points which is physically significant. If potential at infinity is chosen to be zero; potential at a point with position vector $\vec{r}$ due to a point charge Q placed at the origin is given by

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

3. The electrostatic potential at a point with position vector $\vec{r}$ due to a point dipole of dipole moment $\vec{p}$ placed at the origin is

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

The result is true also for a dipole (with charges $-q$ and q separated by $2a$) for $r >> a$.

4. For a charge configuration $q_1, q_2, ..., q_n$ with position vectors $\vec{r}_1, \vec{r}_2, ...\vec{r}_n$, the potential at a point P is given by the superposition principle

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + ... + \frac{q_n}{r_{nP}}\right)$$

where r_{1P} is the difference between q_1 and P, as and so on.

5. An equipotential surface is a surface over which potential has a constant value. For a point charge, concentric spheres centered at a location of the charge are equipotential surfaces. The electric field $\vec{E}$ at a point is perpendicular to the equipotential surface through the point. $\vec{E}$ is in the direction of the steepest decrease of potential.

6. Potential energy stored in a system of charges is the work done (by an external agency) in assembling the charges at their locations. Potential energy of two charges q_1, q_2 at r_1, r_2 is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$ where r_{12} is distance between q_1 and q_2.

7. The potential energy of a charge q in an external potential $V(r)$ is $qV(r)$. The potential energy of a dipole moment $\vec{p}$ in a uniform electric field $\vec{E}$ is $-\vec{p}\cdot\vec{E}$.

8. Electrostatics field $\vec{E}$ is zero in the interior of a conductor; just outside the surface of a charged conductor, $\vec{E}$ is normal to the surface given by $\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{n}$ where $\hat{n}$ is the unit vector along the outward normal to the surface and σ is the surface charge density. Charges in a conductor can reside only at its surface. Potential is constant within and on the surface of a conductor. In a cavity within a conductor (with no charges), the electric field is zero.

9. A capacitor is a system of two conductors separated by an insulator. Its capacitance is defined by $C=Q/V$, where Q and $-Q$ are the charges on the two conductors and V is the potential difference between them. C is determined purely geometrically, by the shapes, sizes and relative positions of the two conductors. The unit of capacitance is farad, $1\text{F}=1\text{CV}^{-1}$. For a parallel plate capacitor (with vacuum between the plates),

$$C=\varepsilon_0\frac{A}{d}$$

10. If the medium between the plates of a capacitor is filled with an insulating substance (dielectric), the electric field due to the charged plates induces a net dipole moment in the dielectric. This effect, called polarization, gives rise to a field in the opposite direction. The net electric field inside the dielectric and hence the potential difference between the plates is thus reduced. Consequently, the capacitance C increases from its value C_0 when there is no medium (vacuum),

$$C=KC_0$$

where K is the dielectric constant of the insulating substance.

11. For capacitors in the series combination, the total capacitance C is given by

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+...$$

In the parallel combination, the total capacitance C is:

$$C=C_1+C_2+C_3+...$$

where $C_1, C_2, C_3...$ are individual capacitances.

12. The energy U stored in a capacitor of capacitance C, with charge Q and voltage V is

$$U=\frac{1}{2}QV=\frac{1}{2}CV^2=\frac{1}{2}\frac{Q^2}{C}$$

The electric energy density (energy per unit volume) in a region with electric field is $(1/2)\varepsilon_0E^2$.

13. A Van de Graaff generator consists of a large spherical conducting shell (a few meters in diameter). By means of a moving belt and suitable brushes, charge is continuously transferred to the shell and potential difference of the order of several million volts is built up, which can be used for accelerating charged particles.

Physical quantity	Symbol	Dimensions	Unit	Remark
Potential	ϕ or V	$\left[M^1L^2T^{-3}A^{-1}\right]$	V	Potential difference is physically significant
Capacitance	C	$\left[M^{-1}L^{-2}T^{-4}A^2\right]$	F	
Polarisation	P	$\left[L^{-2}AT\right]$	$\mathrm{C\,m}^{-2}$	Dipole moment per unit volume
Dielectric constant	K	[Dimensionless]		

Question 1:

(a) Calculate the potential at a point P due to a charge of $4\times10^{-7}C$ located $9\,cm$ away.

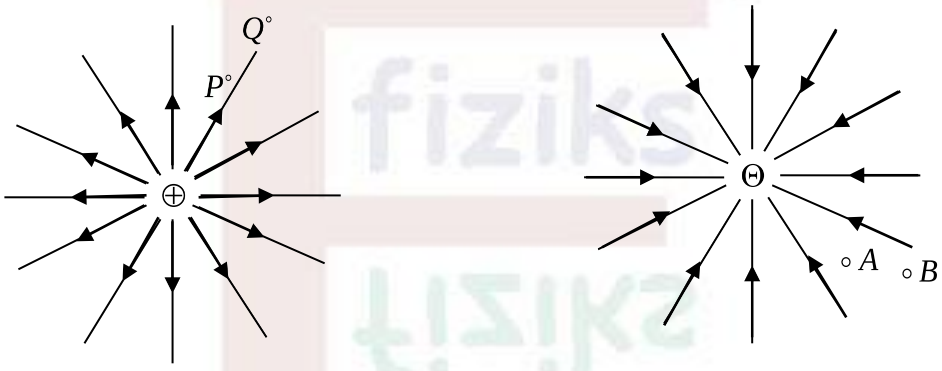
(b) Hence obtain the work done in bringing a charge of $2\times10^{-9}C$ from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Question 2:

Two charges $3\times10^{-8}C$ and $-2\times10^{-8}C$ are located $15\,cm$ apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Question 3:

Figures (a) and (b) show the field lines of a positive and negative point charge respectively.



(a) (b)

(a) Give the signs of the potential difference $V_p - V_Q; V_B - V_A$.

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

Question 4: Two charges $5\times10^{-8}C$ and $-3\times10^{-8}C$ are located $16\,cm$ apart. At what points on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

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Question 5: A regular hexagon of side $10\ cm$ has a charge $5\mu C$ at each of its vertices. Calculate the potential at the center of the hexagon.

Question 6: Two charges $2\mu C$ and $-2\mu C$ are placed at points A and B, $6cm$ apart.

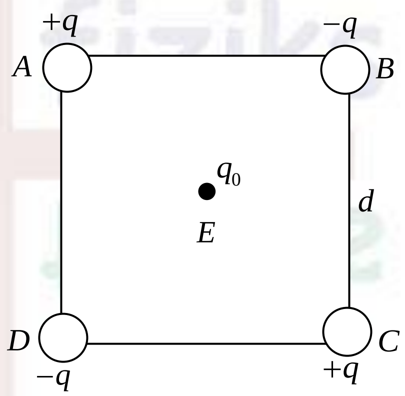
(a) Identify an equipotential surface of the system.

(b) What is the direction of the electric field at every point on this surface?

Question 7:

Four charges are arranged at the corners of a square $ABCD$ of side d, as shown in figure.

(a) Find the work required to put together this arrangement. (b) A charge q_0, is brought to the center E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?



Question 8:

(a) Determine the electrostatic potential energy of a system consisting of two charges $7\ \mu C$ and $-2\ \mu C$ (and with no external field) placed at $(-9cm, 0, 0)$ and $(9cm, 0, 0)$ respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external electric field $E = A\left(1/r^2\right);\ A = 9\times10^5 C\,m^{-2}$. What would the electrostatic energy of the configuration be?

Question 9:

A molecule of a substance has a permanent electric dipole moment of magnitude $10^{-29} Cm$. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude $10^6\ Vm^{-1}$. The direction of the field is suddenly changed by an angle of 60^0. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarization of the sample.

Question 10: A charge of $8\,mC$ is located at the origin. Calculate the work done in taking a small charge of $-2\times10^{-9}C$ from a point $P(0,0,3cm)$ to a point $Q(0,4cm,0)$, via a point $R(0,6cm,9cm)$. Charge located at the origin, $q = 8mC = 8\times10^{-3}C$

Question 11: A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the center of the cube.

Question 12: Two tiny spheres carrying charges $1.5\,\mu C$ and $2.5\,\mu C$ are located $30\,cm$ apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point $10\,cm$ from this midpoint in a plane normal to the line and passing through the mid-point.

Question 13:

(a) A comb run through one's dry hair attracts small bits of paper. Why?

What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity.)

(b) Ordinary rubber is an insulator. But special rubber tires of aircraft are made slightly conducting. Why is this necessary?

(c) Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why?

(d) A bird perches on a bare high-power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

Question 14: A spherical conductor of radius $12\,cm$ has a charge of $1.6\times10^{-7}C$ distributed uniformly on its surface. What is the electric field

(a) Inside the sphere (b) Just outside the sphere

(c) At a point $18\,cm$ from the center of the sphere?

Question 15: A parallel plate capacitor with air between the plates has a capacitance of $8\,pF\left(1\,pF = 10^{-12}F\right)$. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

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Question 16: Three capacitors each of capacitance $9\,pF$ are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a $120V$ supply?

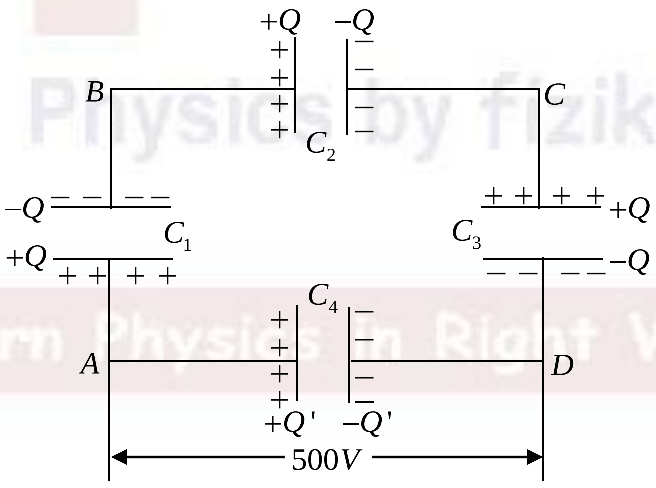
Question 17: Three capacitors of capacitance $2\,pF, 3pF$ and $4\,pF$ are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a $100V$ supply.

Question 18: A slab of material of dielectric constant K has the same area as the plates of a parallel-plate capacitor but has a thickness $(3/4)d$, where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

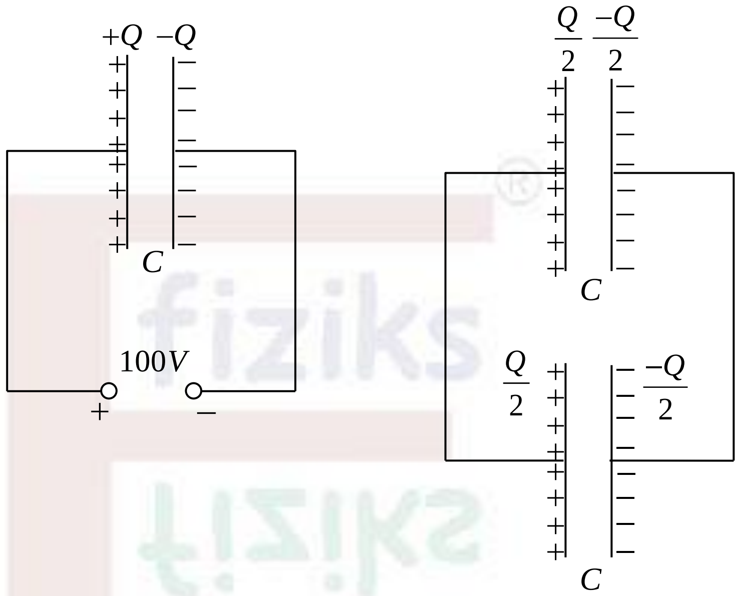
Question 19: A network of four $10\,\mu F$ capacitors is connected to a $500V$ supply, as shown in Fig. 2.29. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor. (Note, the charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.)



Question 20:

(a) A $900\,pF$ capacitor is charged by $100V$ battery [Fig. (a)]. How much electrostatic energy is stored by the capacitor?

(b) The capacitor is disconnected from the battery and connected to another $900\,pF$ capacitor (Fig. (b). What is the electrostatic energy stored by the system?



Question 21: A $600\,pF$ capacitor is charged by a $200V$ supply. It is then disconnected from the supply and is connected to another uncharged $600\,pF$ capacitor. How much electrostatic energy is lost in the process?

Question 22: A spherical capacitor has an inner sphere of radius $12\,cm$ and an outer sphere of radius $13\,cm$. The outer sphere is earthed and the inner sphere is given a charge of $2.5\,\mu C$. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere?

(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Question 23:

A cylindrical capacitor has two co-axial cylinders of length $15\,cm$ and radii $1.5\,cm$ and $1.4\,cm$. The outer cylinder is earthed and the inner cylinder is given a charge of $3.5\,\mu C$. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).