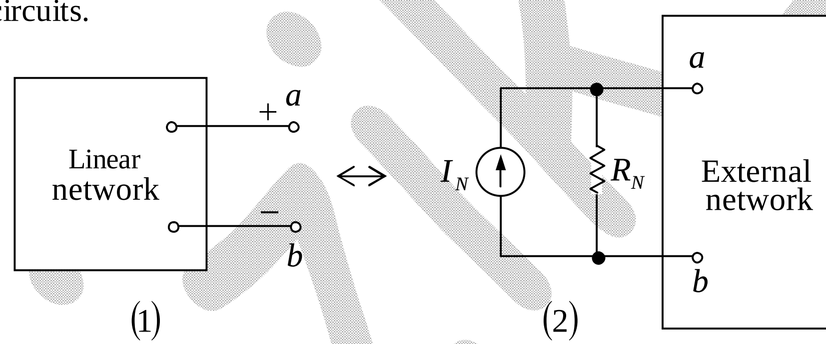
1(d). Norton's Theorem

Norton's theorem is used to simplify a network in terms of currents instead of voltages. For current analysis, this theorem can be used to reduce a network to a simple parallel circuit with a current source, which supplies a total line current that can be divided among parallel branches.

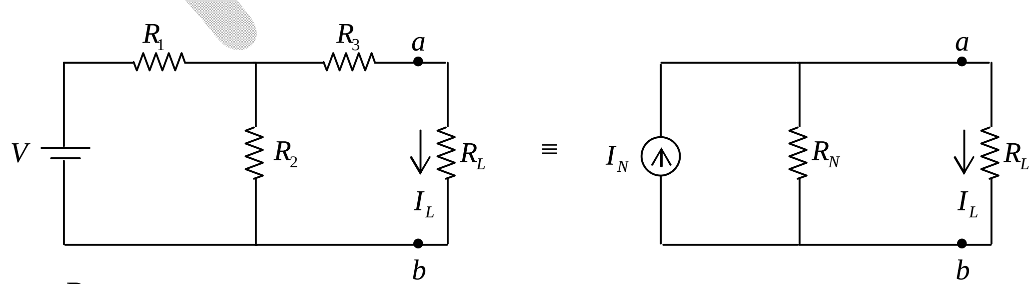
Norton's theorem states that any network connected to terminals a and b [figure (1)] can be replaced by a single current source I_N in parallel with a single resistance R_N [figure (2)]. I_N is equal to the short-circuit current through the $a\,b$ terminals (the current that the network would produce through a and b with a short circuit across these two terminals). The value of the single resistor is the same for both the Norton and Thevenin equivalent circuits.



(a) Norton equivalent, I_N and parallel R_N

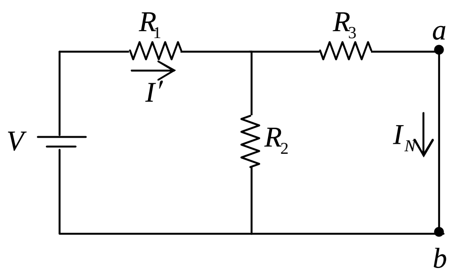
This direction must be the same as the current produced by the polarity of the corresponding voltage source. Remember that a source produces current flow out from the positive terminal.

Consider two loop network as shown in figure.



Then $I_L = \dfrac{R_N}{R_N + R_L} I_N$

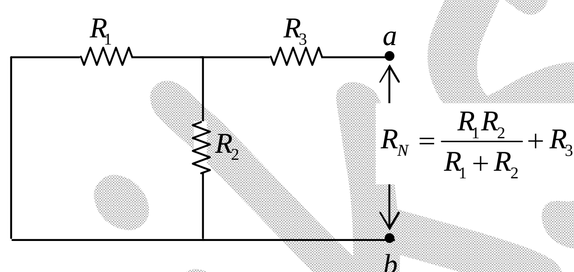
Norton Current (I_N)



$$I' = \frac{V}{R_1 + R_2R_3/(R_2+R_3)}$$

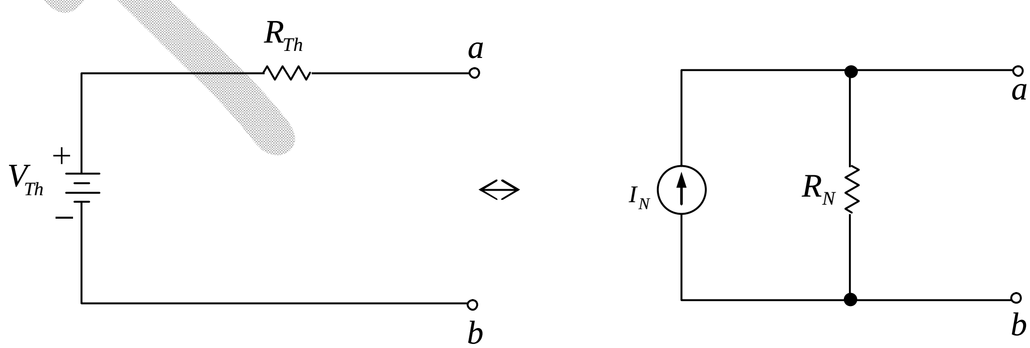
$$I_N = \frac{R_2}{R_2+R_3}I'$$

Norton Resistance (R_N)



We therefore see that the Thevenin equivalent circuit (figure a) corresponds to the Norton equivalent circuit (figure b). So a general voltage source with a series resistance can be converted to an equivalent current source with the same resistance in parallel. Divide the general source V by its series resistance R to find the value of I for the equivalent current source shunted by the same resistance R; that is $I_N = \frac{V_{Th}}{R_{Th}}$.

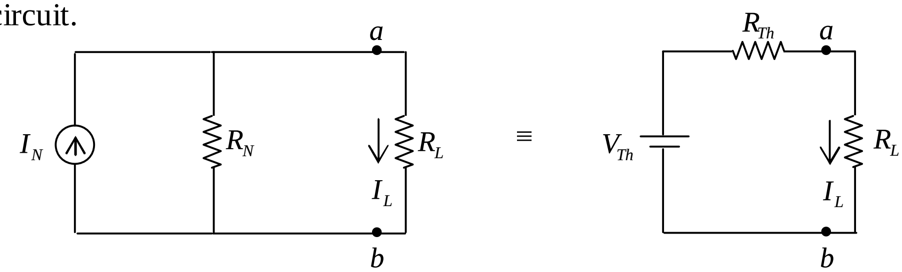
We therefore see that Thevenin equivalent circuit (figure a) corresponds to the Norton equivalent circuit (figure b).



(a) Thevenin circuit (b) Norton circuit

Figure: Equivalent circuits

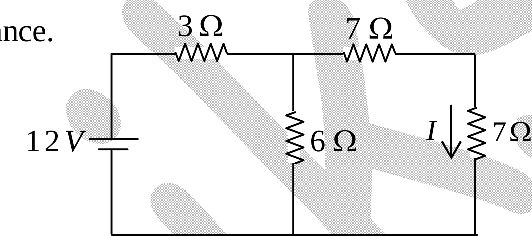
We can also see that the Norton equivalent circuit corresponds to the Thevenin equivalent circuit.



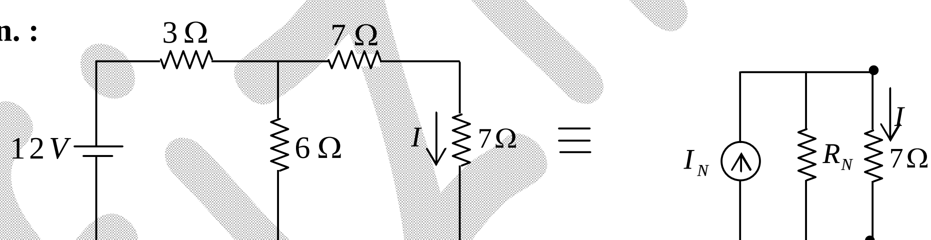
where $V_{Th} = I_N R_N$ (open circuit voltage across ab)

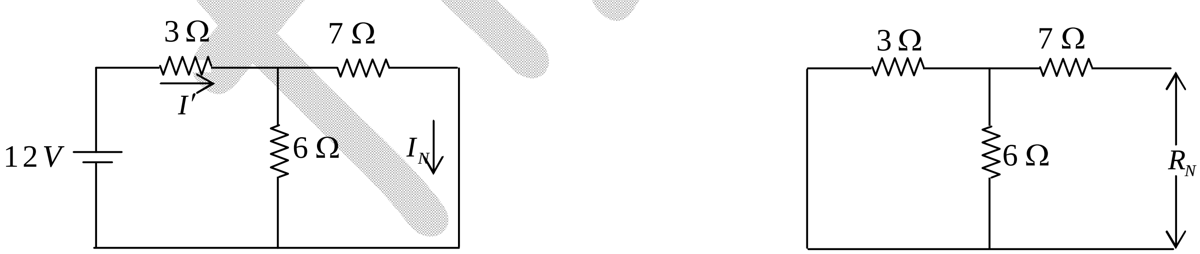
and $R_{Th} = R_N$ (open the current source and measure the equivalent resistance across ab)

Example: Draw Norton equivalent circuit for the circuit shown in figure below and find current I across $7\,\Omega$ resistance.



Solution. :





$$I' = \frac{12}{3 + \frac{6\times 7}{6+7}} = \frac{52}{27}A,\ I_N = \frac{6}{6+7}\times\frac{52}{27} = \frac{8}{9}A \qquad R_N = \frac{3\times 6}{3+6} + 7 = 9\,\Omega$$

Thus $I = \frac{R_N}{R_N + 7} I_N = \frac{9}{9+7}\times\frac{8}{9} = \frac{1}{2}A$