(f) The Fermi Level

Electrons in solids obey *Fermi-Dirac* statistics. In the development of this type of statistics, one must consider the indistinguishability of the electrons, their wave nature, and the Pauli Exclusion Principle. The rather simple result of these statistical arguments is that the distribution of electrons over a range of allowed energy levels at thermal equilibrium is:

$$f(E)=\frac{1}{1+e^{(E-E_F)/kT}}$$

where k is Boltzmann constant. The function $f(E)$, the *Fermi-Dirac distribution function,* gives the probability that an available energy state at E will be occupied by an electron at absolute temperature T. The quantity E_F is called the *Fermi Level,* and it represents an important quantity in the analysis of semiconductor behavior. We notice that, for an energy E equal to the Fermi level energy E_F, the occupation probability is

$$f(E_F)=\left[1+e^{(E_F-E_F)/kT}\right]^{-1}=\frac{1}{1+1}=\frac{1}{2}.$$

A closer examination of $f(E)$ indicates that at $0\,K$ the distribution takes the simple rectangular form shown in figure. With $T = 0$ in the denominator of the exponent, $f(E)$ is $1/(1 + 0) = 1$ when the exponent is negative ($E < E_F$), and is $1/(1 + \infty) = 0$ when the exponent is positive ($E > E_F$). This rectangular distribution implies that at $0\,K$ every available energy state up to E_F is filled with electrons and all states above E_F are empty.

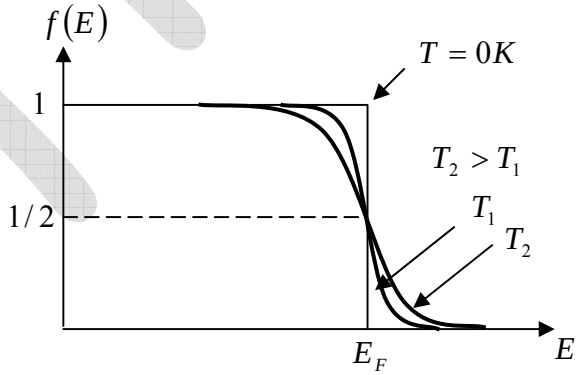

Figure (i): The Fermi Dirac distribution function.

At temperatures higher than $0\,K$, some probability exists for states above E_F to be filled. For example, at $T = T_1$ there is some probability $f(E)$ that states above E_F are filled, and there is a corresponding probability $[1 - f(E)]$ that states below E_F are empty.

The Fermi function is symmetrical about E_F for all temperatures; that is the probability $f(E_F + \Delta E)$ that a state ΔE above E_F is filled is the same as the probability $[1 - f(E_F - \Delta E)]$ that a state ΔE below E_F is empty. The symmetry of the distribution of empty and filled states about E_F makes the Fermi level a natural reference point in calculations of electron and hole concentrations in semiconductors.

For intrinsic material we know that the concentration of holes in the valence band is equal to the concentration of electrons in the conduction band. Therefore, the Fermi level E_F must lie at the middle of the band gap in intrinsic material [Figure ii (a)]. Since $f(E)$ is symmetrical about E_F, the electron probability "tail" of $f(E)$ extending into the conduction band is symmetrical with the hole probability tail $[1 - f(E)]$ in the valence band. The distribution function has values within the band gap between E_c and E_v but there are no energy states available, and no electron occupancy results from $f(E)$ in this range.

In *n*-type material there is a high concentration of electrons in the conduction band compared with the hole concentration in the valence band. Thus in *n*-type material the distribution function $f(E)$ must lie above its intrinsic position on the energy scale [figure ii (b)]. Since $f(E)$ retains its shape for a particular temperature, the larger concentration of electrons at E_c in *n*-type material implies a correspondingly smaller hole concentration at E_v. We notice that the value of $f(E)$ for each energy level in the conduction band (and therefore the total electron concentration n_0) increases as E_F moves closer to E_c. Thus the energy difference $(E_c - E_F)$ gives a measure of n.

For *p*-type material the Fermi level lies near the valence band [figure ii (c)] such that the $[1 - f(E)]$ tail below E_v is larger than the $f(E)$ tail above E_c. The value of $(E_F - E_v)$

indicates how strongly *p*-type the material is.

It is usually inconvenient to draw $f(E)$ *vs.* E on every energy band diagram to indicate the electron and hole distributions. Therefore, it is common practice merely to indicate the position of E_F in band diagrams.

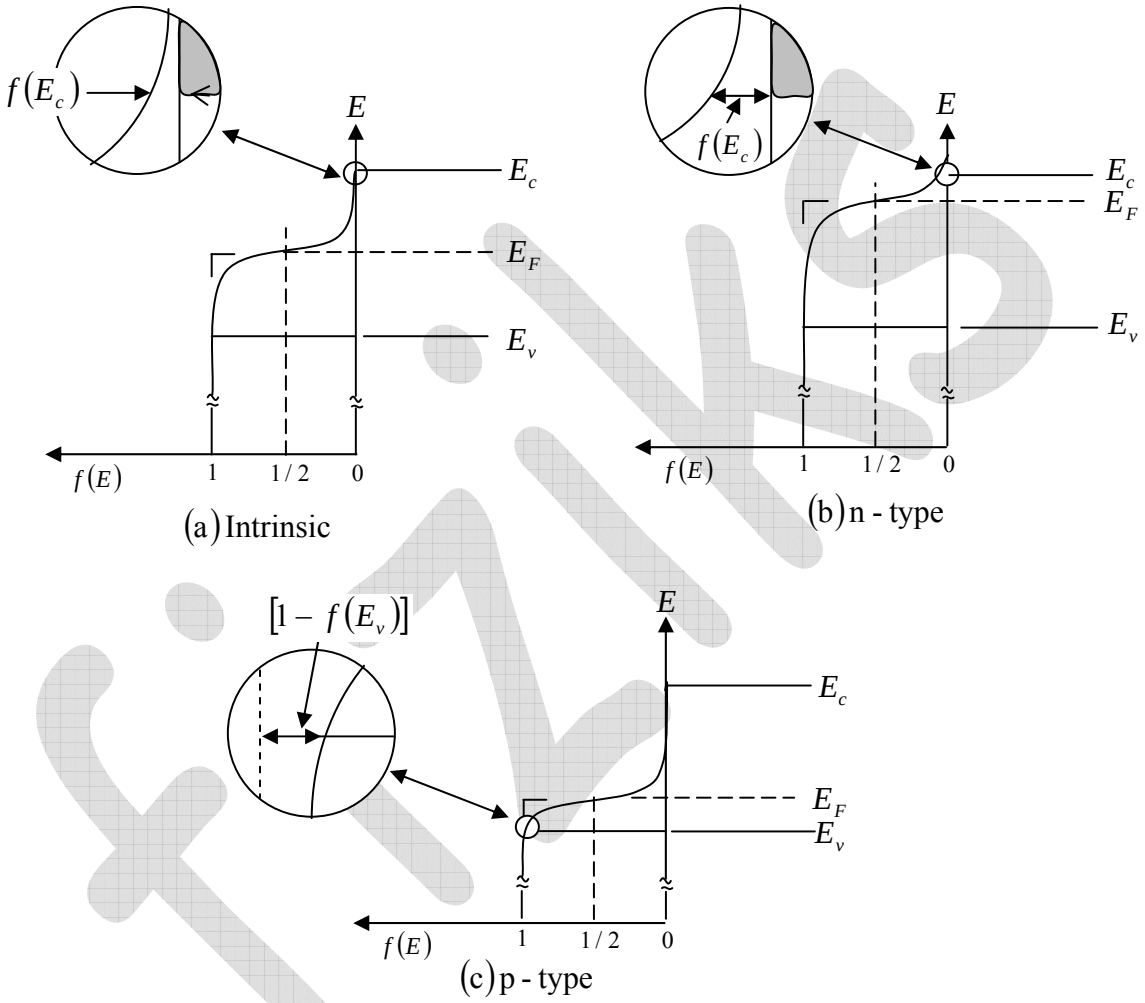

Figure (ii) : The Fermi distribution function applied to semiconductors:
(a) Intrinsic material; (b) *n*-type material; (c) *p*-type material.