

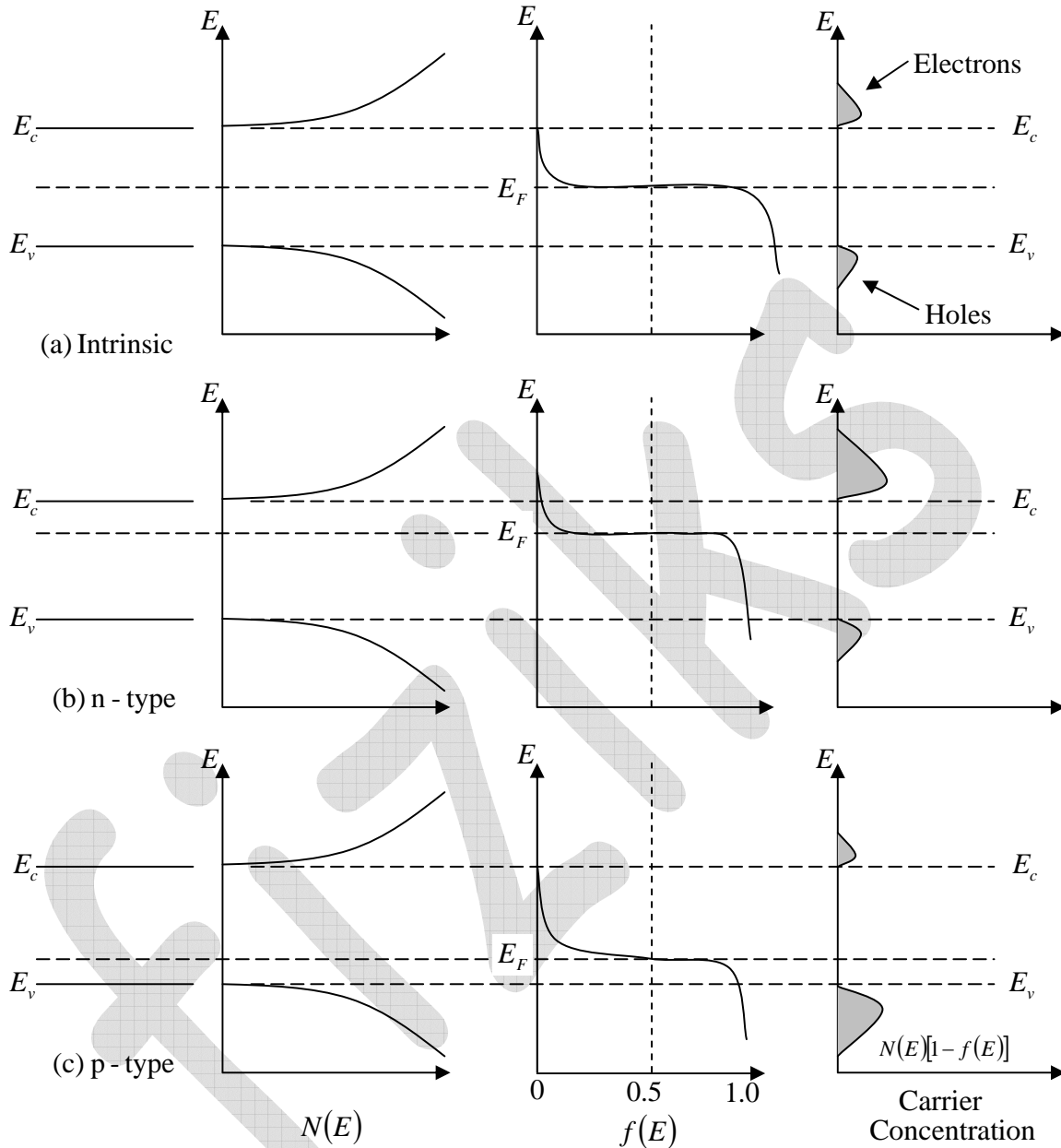
### (g) Electron and Hole Concentrations at Equilibrium

The Fermi distribution function can be used to calculate the concentrations of electrons and holes in a semiconductor, if the densities of available states in the valence and conduction bands are known. For example, the concentration of electrons in the conduction band is

$$n_0 = \int_{E_c}^{\infty} f(E) N(E) dE$$

where  $N(E)dE$  is the density of states ( $\text{cm}^{-3}$ ) in the energy range  $dE$ . The subscript 0 used with the electron and hole concentration symbols ( $n_0, p_0$ ) indicates equilibrium conditions. The number of electrons per unit volume in the energy range  $dE$  is the product of the density of states and the probability of occupancy  $f(E)$ . Thus the total electron concentration is the integral over the entire conduction band. The function  $N(E)$  can be calculated by using quantum mechanics and the Pauli Exclusion Principle.

Since  $N(E)$  is proportional to  $E^{1/2}$ , so the density of states in the conduction band increases with electron energy. On the other hand, the Fermi function becomes extremely small for large energies. The result is that the product  $f(E)N(E)$  decreases rapidly above  $E_c$  and very few electrons occupy energy states far above the conduction band edge. Similarly, the probability of finding an empty state (hole) in the valence band  $[1 - f(E)]$  decreases rapidly below  $E_v$  and most holes occupy states near the top of the valence band. This effect is demonstrated in figure, which shows the density of available states, the Fermi function, and the resulting number of electrons and holes occupying available energy states in the conduction and valence bands at thermal equilibrium (i.e., with no excitations except thermal energy). For holes, increasing energy points down, since the  $E$  scale refers to electron energy.



**Figure:** Schematic band diagram, density of states, Fermi-Dirac distribution, and the carrier concentrations for (a) intrinsic, (b) n-type, and (c) p-type semiconductors at thermal equilibrium.

The result of the integration of  $n_0 = \int_{E_c}^{\infty} f(E)N(E)dE$  is the same as that obtained if we represent the entire distributed electron states in the conduction band by an *effective density of states*  $N_c$  located at the conduction band edge  $E_c$ . Therefore, the conduction band electron

concentration is simply the effective density of states at  $E_c$  times the probability of occupancy at  $E_c$

$$n_0 = f(E_c)N_c$$

In this expression we assume the Fermi level  $E_F$  lies at least several  $kT$  below the conduction band. Then the exponential term is large compared with unity and the Fermi function  $f(E_c)$  can be simplified as

$$f(E_c) = \frac{1}{1 + e^{(E_c - E_F)/kT}} \approx e^{-(E_c - E_F)/kT}$$

Since  $kT$  at room temperature is only  $0.026 \text{ eV}$ , this is generally a good approximation. For this condition the concentration of electrons in the conduction band is

$$n_0 = N_c e^{-(E_c - E_F)/kT} \Rightarrow E_c - E_F = kT \ln \left( \frac{N_c}{n_0} \right)$$

The effective density of states  $N_c = 2 \left( \frac{2\pi m_n^* kT}{h^2} \right)^{3/2}$

Thus electron concentration increases as  $E_F$  moves closer to the conduction band.

By similar arguments, the concentration of holes in the valence band is

$$p_0 = N_v [1 - f(E_v)]$$

where  $N_v$  is the effective density of states in the valence band.

The probability of finding an empty state at  $E_v$  is,

$$1 - f(E_v) = 1 - \frac{1}{1 + e^{-(E_v - E_F)/kT}} \approx e^{-(E_F - E_v)/kT} \text{ for } E_F \text{ larger than } E_v \text{ by several } kT.$$

From these equations, the concentration of holes in the valence band is

$$p_0 = N_v e^{-(E_F - E_v)/kT} \Rightarrow E_F - E_v = kT \ln \left( \frac{N_v}{p_0} \right)$$

The effective density of states in the valence band  $N_v = 2 \left( \frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$

Thus hole concentration increases as  $E_F$  moves closer to the valence band.

The electron and hole concentrations predicted by above equations are valid whether the material is intrinsic or doped, provided thermal equilibrium is maintained.

Thus for *intrinsic material*,  $E_F$  lies, at some intrinsic level  $E_i$  near the middle of the band gap, and the intrinsic electron and hole concentrations are

$$n_i = N_c e^{-(E_c - E_i)/kT}, \quad p_i = N_v e^{-(E_i - E_v)/kT}$$

$$n_i = p_i \Rightarrow E_i = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln \left( \frac{N_v}{N_c} \right) \Rightarrow E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln \left( \frac{m_p^*}{m_n^*} \right)$$

**Note:** The intrinsic level  $E_i$  is the middle of the band gap  $\left( E_c - E_i = \frac{E_g}{2} \right)$ , if the effective densities of states  $N_c$  and  $N_v$  are equal. There is usually some difference in effective mass for electrons and holes, however, and  $N_c$  and  $N_v$  are slightly different.

The product of  $n_0$  and  $p_0$  at equilibrium is a constant for a particular material and temperature, even if the doping is varied:

$$n_0 p_0 = \left( N_c e^{-(E_c - E_F)/kT} \right) \left( N_v e^{-(E_F - E_v)/kT} \right) = N_c N_v e^{-(E_c - E_v)/kT} = N_c N_v e^{-E_g/kT}$$

$$n_i p_i = \left( N_c e^{-(E_c - E_i)/kT} \right) \left( N_v e^{-(E_i - E_v)/kT} \right) = N_c N_v e^{-E_g/kT}$$

The intrinsic electron and hole concentrations are equal (since the carriers are created in pairs),  $n_i = p_i$ ; thus the intrinsic concentration is

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

### Law of Mass Action

The constant product of electron and hole concentrations can be written conveniently as

$$n_0 p_0 = n_i^2$$

For  $n$ -type material the minority concentration (holes)

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D} \quad \text{where } N_D \text{ is donor ion concentration.}$$

For  $p$ -type material the minority concentration (electrons)

$$n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A} \quad \text{where } N_A \text{ is acceptor ion concentration.}$$

Another convenient way of writing electron and hole concentration is

$$n_0 = n_i e^{(E_F - E_i)/kT} \quad \text{and} \quad p_0 = n_i e^{(E_i - E_F)/kT}$$

This form of the equation indicates directly that the electron concentration is  $n_i$  when  $E_F$  is at the intrinsic level  $E_i$  and that  $n_0$  increases exponentially as the Fermi level moves away from  $E_i$  toward the conduction band. Similarly, the hole concentration  $p_0$  varies from  $n_i$  to larger values as  $E_F$  moves from  $E_i$ , toward the valence band. Since these equations reveal the qualitative features of carrier concentration so directly, they are particularly convenient to remember.