(j) Drift Current (Conductivity and Mobility)

The charge carriers in a solid are in constant motion, even at thermal equilibrium. At room temperature, for example, the thermal motion of an individual electron may be visualized as random scattering from lattice atoms, impurities, other electrons, and defects (figure). Since the scattering is random, there is no net motion of the group of $n\ electrons/cm^3$ over any period of time. This is not true of an individual electron, of course. The probability of the electron in returning to its starting point after some time t is negligibly small. However, if a large number of electrons is considered (e.g. $10^{16}\ cm^{-3}$ in an n-type semiconductor), there will be no preferred direction of motion for the group of electrons and no net current flow.

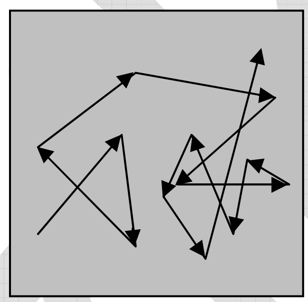

Figure: Thermal motion of an electron in a solid.

If an electric field E_x is applied in the x-direction, each electron experiences a net force $-qE_x$ from the field. This force may be insufficient to alter appreciably the random path of an individual electron; the effect when averaged over all the electrons, however, is a net motion of the group in the x-direction. If p_x is the x-component of the total momentum of the group, force of the field on the $n\ electrons/cm^3$ is

$$-nqE_x = \left.\frac{dp_x}{dt}\right|_{\text{field}} \quad .$$

Initially, above equation seems to indicate a continuous acceleration of the electrons in the $-x$-direction. This is not the case, however, because the net acceleration is just balanced in steady state by the decelerations of the collision processes. Thus while the steady field E_x does produce a net momentum p_{-x}, the net rate of change of momentum when collisions are included must be zero in the case of steady state current flow.

To find the total rate of momentum change from collisions, we must investigate the collision probabilities more closely. If the collisions are truly random, there will be a constant probability of collision at any time for each electron. Let us consider a group of N_0 electrons at time $t=0$ and define $N(t)$ as the number of electrons that have not undergone a collision by time *t*. The rate of decrease in $N(t)$ at any time *t* is proportional to the number left unscattered at *t*,

$$-\frac{dN(t)}{dt}=\frac{1}{\bar{t}}N(t)$$ where $\bar{t}^{-1}$ is a constant proportionality.

The solution to above equation is an exponential function $N(t)=N_0e^{-t/\bar{t}}$ and $\bar{t}$ represents the mean time between scattering events, called the ***mean free time***.

The probability that any electron has a collision in the time interval dt is $\frac{dt}{\bar{t}}$.

Thus the differential change in *p*, due to collisions in time dt is $dp_x=-p_x\frac{dt}{\bar{t}}$.

The rate of change of p_x, due to the decelerating effect of collisions is $\left.\frac{dp_x}{dt}\right|_{\text{collision}}=-\frac{p_x}{\bar{t}}$

The sum of acceleration and deceleration effects must be zero for steady state. Thus

$$-\frac{p_x}{\bar{t}}-nqE_x=0.$$

The average momentum per electron is $<p_x>=\frac{p_x}{n}=-q\bar{t}E_x$ where the angular brackets indicate an average over the entire group of electrons. As expected for steady state, the above equation indicates that the electrons have *on the average* a constant net velocity in the negative *x*-direction:

$$\langle v_x\rangle=\frac{\langle p_x\rangle}{m_n^*}=-\frac{q\bar{t}}{m_n^*}E_x$$

Actually, the individual electrons move in many directions by thermal motion during a given time period, but $\langle v_x\rangle$ tells us the *net* ***drift*** of an average electron in response to the electric field. The drift speed $\langle v_x\rangle$ is usually much smaller than the random speed due to thermal motion v_{th}.

The **current density** resulting from this net drift is just the number of electrons crossing a unit area per unit time $\left(n\langle v_x\rangle\right)$ multiplied by the charge on the electron $(-q)$:

$$J_x = -qn\langle v_x\rangle = \frac{nq^2\bar{t}}{m_n^*}E_x \quad ampere/cm^2 .$$

Thus the current density is proportional to the electric field, as we expect from Ohm's law: $J_x = \sigma E_x$ where $\sigma \equiv \frac{nq^2\bar{t}}{m_n^*}$.

The **conductivity** $\sigma(\Omega - cm)^{-1}$ can be written $\boxed{\sigma = qn\mu_n}$ where $\mu_n \equiv \frac{q\bar{t}}{m_n^*}$.

The quantity μ_n, called the ***electron mobility,*** describes the ease with which electrons drift in the material. Mobility is a very important quantity in characterizing semiconductor materials and in device development.

The mobility can be expressed as the average particle drift velocity per unit electric field.

Thus $\mu_n = -\frac{\langle v_x\rangle}{E_x}$, and units of mobility are $(cm/s)/(V/cm) = cm^2/V\text{-}s$. The minus sign in the definition results in a positive value of mobility, since electrons drift opposite to the field.

The current density can be written in terms of mobility as $J_x = qn\mu_n E_x$.

This derivation has been based on the assumption that the current is carried primarily by electrons. For hole conduction we change n to p, $-q$ to $+q$, μ_n to μ_p

where $\mu_p = +\frac{\langle v_x\rangle}{E_x}$ is the mobility for holes.

If both electrons and holes participate, then

where $\boxed{\sigma = q\left(n\mu_n + p\mu_p\right)}$.

For N-type semiconductor

$\sigma_n = e\left(n_n\mu_n + p_n\mu_p\right) \approx n_n e\mu_n$ since $n_n >> p_n$ where n_n and p_n are electron and hole concentration in *N*-type.

For P-type semiconductor

$\sigma_p = e\left(n_p\mu_n + p_p\mu_p\right) \approx p_p e\mu_p$ since $p_p >> n_p$ where n_p and p_p are electron and hole concentration in *P*-type.