(b) Orbital and Spin Magnetic Moment

An electron is a charged particle and there is a magnetic moment associated with its angular momentum. because the electron in an atom may have two types of angular momentum, spin and orbital angular momentum, there are two sources of magnetic moment. These two magnetic moments can interact and give rise to shifts in the energies of the states of the atom which affect the appearance of the spectrum of the atom. The resulting shifts and splitting of lines is called the fine structure of the spectrum.

(i) Orbital Magnetic Dipole Moment:

Consider an electron of mass m and charge $-e$ moving with velocity of magnitude v in a circular Bohr orbit of radius r as shown in figure (a). The charge circulating in a loop constitutes a current of magnitude

$$I = -\frac{e}{T} = -\frac{ev}{2\pi r}$$

Where, T is the orbital period of the electron. Such current loop produces a magnetic field which is identical to fictitious magnetic dipole moment that produces similar field far from the loop. For a current I in a loop of area A, the magnitude of orbital magnetic dipole moment is

$$\mu_L = IA$$

Because of negative charge, the magnetic dipole moment is anti-parallel to its orbital angular momentum $L = mvr$. Thus

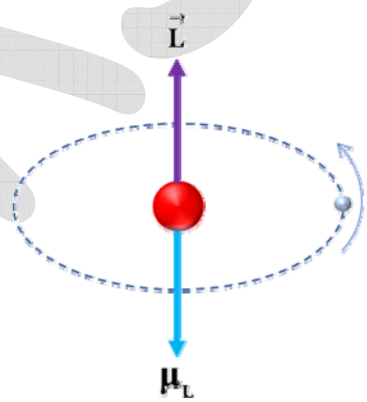

Figure (a): Orbital angular momentum and orbital magnetic moment of the electron

$$\mu_L = IA = -\frac{ev}{2\pi r} r^2 = -\frac{evr}{2} = -\frac{e}{2m} mvr = -\frac{e}{2m}\mathbf{L}$$

This can be written in term of Bohr magneton $\mu_B = \frac{e\hbar}{2m}$ and is often regarded as the elementary unit of magnetic moment. Its value is $9.27\times 10^{-24}\,\mathrm{J\,T^{-1}}$.

$$|\boldsymbol{\mu}_\mathbf{L}| = -\frac{e\hbar}{2m}\frac{|\mathbf{L}|}{\hbar} = -\mu_B\sqrt{l(l+1)}$$

The z-component of the orbital angular momentum of the electron is

$$\mu_z = -\frac{e}{2m} L_z = \gamma L_z$$

Where $\gamma = -\frac{e}{2m}$. The constant γ is called the gyro-magnetic ratio of the electron.

The properties of the orbital magnetic moment m follow from those of the angular momentum itself. In particular, its z-component is quantized and restricted to the values

$$\mu_{L_z} = \gamma m_l \hbar \qquad m_l = l, l-1, ..., -l$$

Where, the Bohr Magnetron is also defined as

$$\mu_B = -\gamma m_l \hbar = \frac{e\hbar}{2m}$$

In term of Bohr Magneton, the z-component of orbital magnetic moment is

$$\mu_{L_z} = -\mu_B m_l$$

(ii) Spin Magnetic Moment:

Now we consider the magnetic moment that arises from the spin of the electron. By analogy with the orbital magnetic moment, we might expect the spin magnetic moment to be related to the spin angular momentum by $\boldsymbol{\mu}_s = \gamma \mathbf{S}$, but this turns out not to be the case. This should not be too surprising however, because spin has no classical analogue, yet here we are trying to argue by analogy with orbital angular momentum, which does have a classical analogue.

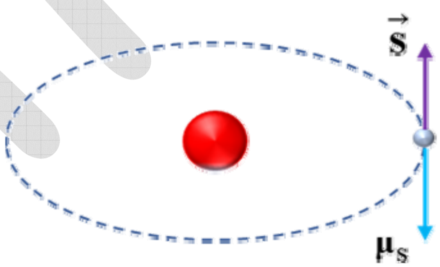

Figure (b): Spin angular momentum and spin magnetic moment of the electron

The relation between the spin and its magnetic moment can be derived from the relativistic Dirac equation, which gives $\mu_s = 2\gamma \mathbf{S}$: the magnetic moment due to spin is twice the value expected on the basis of a classical analogy. The experimental value of the magnetic moment can be determined by observing the effect of a magnetic field on the motion of an electron beam, and it is found that

$$\mu_s = g\gamma \mathbf{S} \quad \text{where} \quad g = 2.002319$$

The factor g is called the g-factor of the electron. The small discrepancy between the experimental value and the Dirac value of exactly 2 is accounted for by the more sophisticated theory of quantum electrodynamics, in which charged particles are allowed to interact with the quantized electromagnetic field. Thus, the magnetic moment due to electron spin can be written as

$$\mu_s = -g\frac{e}{2m}\mathbf{S}$$

The magnitude of $\mathbf{S}$ is equal to $\sqrt{s(s+1)}\,\hbar$, here s is called spin quantum number. Therefore, the absolute value the spin magnetic moment is

$$|\mu_s| = g\mu_B\sqrt{s(s+1)} \cong \sqrt{3}\mu_B$$

Thus, the spin magnetic moment of electron is nearly equal to $\sqrt{3}$ times the Bohr magneton. As for the orbital magnetic moment, the spin magnetic moment has quantized components on the z-axis, and we write

$$\mu_{S_z} = -g\frac{e}{2m}\mathbf{S}_z = -g\mu_B m_s \quad \text{where } m_s = \pm\frac{1}{2}$$

Note: For the electron spin, the ratio of magnetic moment to mechanical angular momentum is twice as large as for the orbital angular momentum of the electron.