2.5 Logarithms

Positive real numbers: x, y, a, c, k
Natural number: n

104. Definition of Logarithm
$y = \log_a x$ if and only if $x = a^y$, $a > 0$, $a \neq 1$.

105. $\log_a 1 = 0$

106. $\log_a a = 1$

107. $\log_a 0 = \begin{cases} -\infty & \text{if } a > 1 \\ +\infty & \text{if } a < 1 \end{cases}$

108. $\log_a (xy) = \log_a x + \log_a y$

109. $\log_a \dfrac{x}{y} = \log_a x - \log_a y$

110. $\log_a\left(x^n\right) = n\log_a x$

111. $\log_a \sqrt[n]{x} = \dfrac{1}{n}\log_a x$

112. $\log_a x = \dfrac{\log_c x}{\log_c a} = \log_c x \cdot \log_a c$, $c > 0$, $c \neq 1$.

113. $\log_a c = \dfrac{1}{\log_c a}$

114. $x = a^{\log_a x}$

115. Logarithm to Base 10
$\log_{10} x = \log x$

116. Natural Logarithm
$\log_e x = \ln x$,

where $e = \lim\limits_{k\to\infty}\left(1 + \dfrac{1}{k}\right)^k = 2.718281828\ldots$

117. $\log x = \dfrac{1}{\ln 10}\ln x = 0.434294\,\ln x$

118. $\ln x = \dfrac{1}{\log e}\log x = 2.302585\,\log x$