# 7.10 Straight Line in Space

Point coordinates: $x, y, z, x_1, y_1, z_1, \ldots$
Direction cosines: $\cos\alpha$, $\cos\beta$, $\cos\gamma$
Real numbers: $A, B, C, D, a, b, c, a_1, a_2, t, \ldots$
Direction vectors of a line: $\vec{s}$, $\vec{s}_1$, $\vec{s}_2$
Normal vector to a plane: $\vec{n}$
Angle between two lines: $\varphi$

**690.** Point Direction Form of the Equation of a Line

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c},$$

where the point $P_1(x_1, y_1, z_1)$ lies on the line, and $(a, b, c)$ is the direction vector of the line.

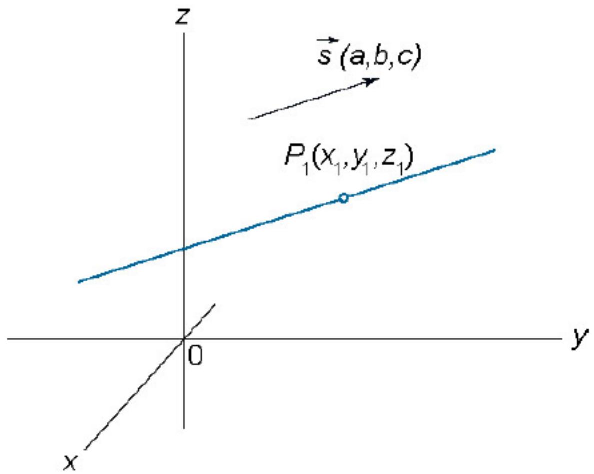


Figure 136.

**691.** Two Point Form

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

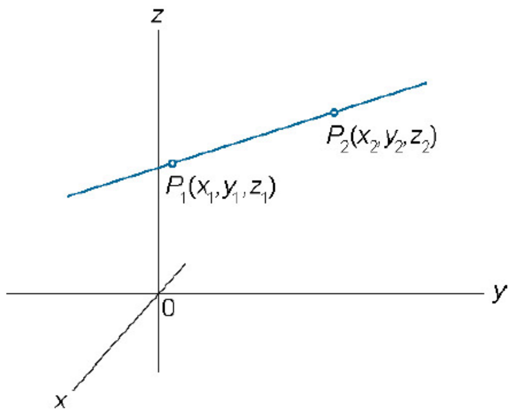


**Figure 137.**

**692.** Parametric Form

$$\begin{cases} x=x_1+t\cos\alpha \\ y=y_1+t\cos\beta \\ z=z_1+t\cos\gamma \end{cases},$$

where the point $P_1(x_1,y_1,z_1)$ lies on the straight line, $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosines of the direction vector of the line, the parameter t is any real number.

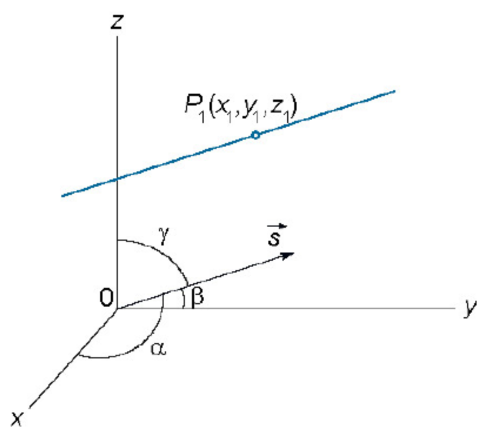


**Figure 138.**

**693.** Angle Between Two Straight Lines

$$\cos\varphi = \frac{\vec{s}_1 \cdot \vec{s}_2}{|\vec{s}_1| \cdot |\vec{s}_2|} = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \cdot \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

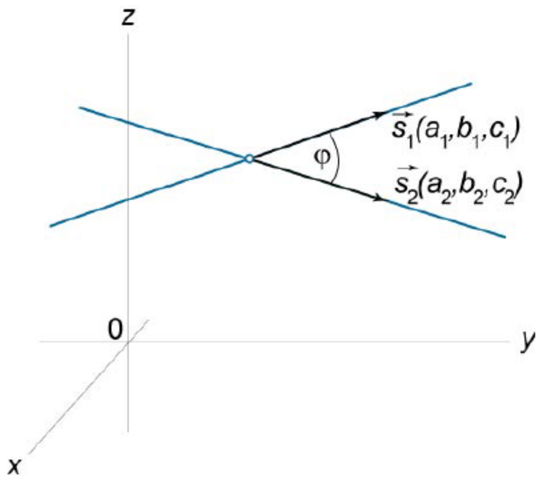


**Figure 139.**

**694.** Parallel Lines

Two lines are parallel if

$\vec{s}_1 \parallel \vec{s}_2$,

or

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}.$$

**695.** Perpendicular Lines

Two lines are parallel if

$\vec{s}_1 \cdot \vec{s}_2 = 0$,

or

$a_1a_2 + b_1b_2 + c_1c_2 = 0$.

**696.** Intersection of Two Lines

Two lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and

$\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ intersect if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0.$$

**697.** Parallel Line and Plane

The straight line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ and the plane $Ax+By+Cz+D=0$ are parallel if

$\vec{n}\cdot\vec{s}=0$,

or

$Aa+Bb+Cc=0$.

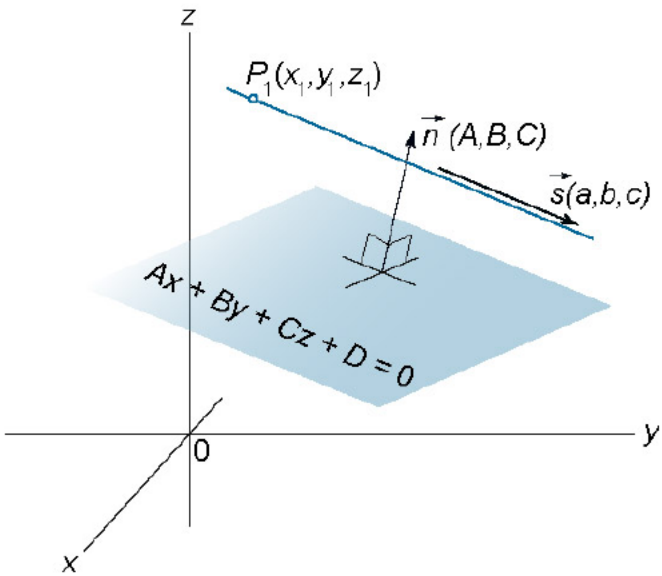


**Figure 140.**

**698.** Perpendicular Line and Plane

The straight line $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ and the plane $Ax+By+Cz+D=0$ are perpendicular if

$\vec{n} \| \vec{s}$,

or

$\frac{A}{a}=\frac{B}{b}=\frac{C}{c}$.

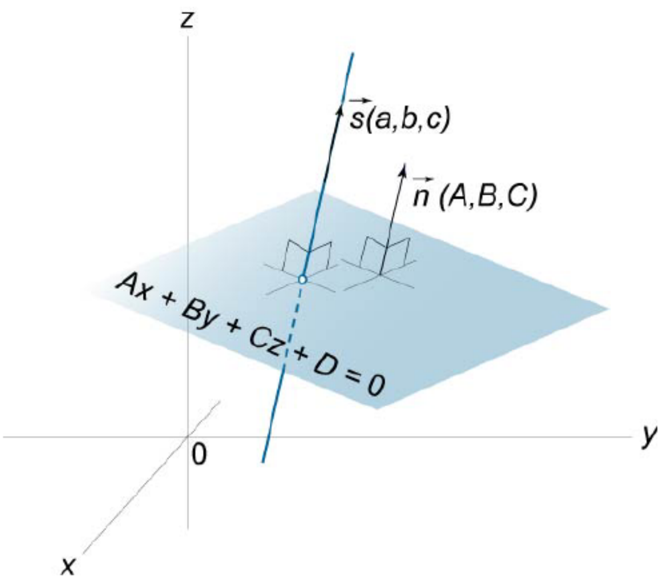


Figure 141.