

7.8 Three-Dimensional Coordinate System

Point coordinates: $x_0, y_0, z_0, x_1, y_1, z_1, \dots$

Real number: λ

Distance between two points: d

Area: S

Volume: V

670. Distance Between Two Points

$$d = AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

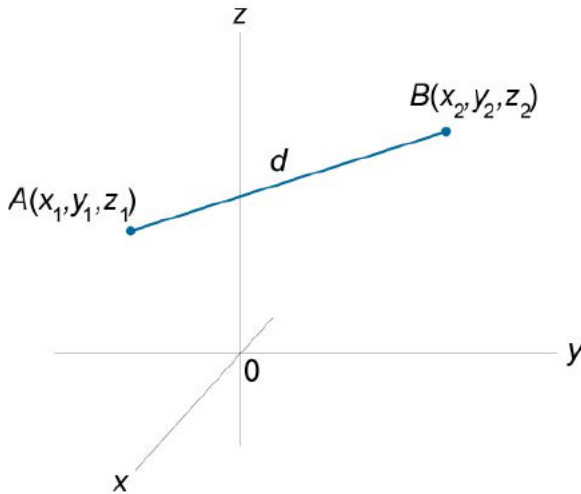


Figure 123.

671. Dividing a Line Segment in the Ratio λ

$$x_0 = \frac{x_1 + \lambda x_2}{1 + \lambda}, \quad y_0 = \frac{y_1 + \lambda y_2}{1 + \lambda}, \quad z_0 = \frac{z_1 + \lambda z_2}{1 + \lambda},$$

where

$$\lambda = \frac{AC}{CB}, \quad \lambda \neq -1.$$

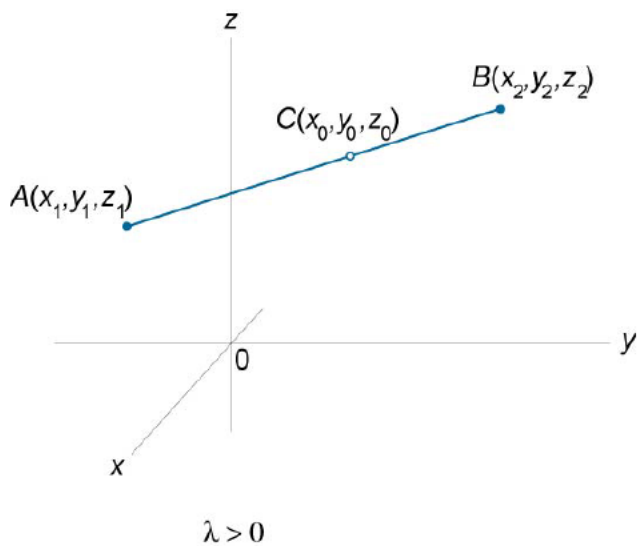


Figure 124.

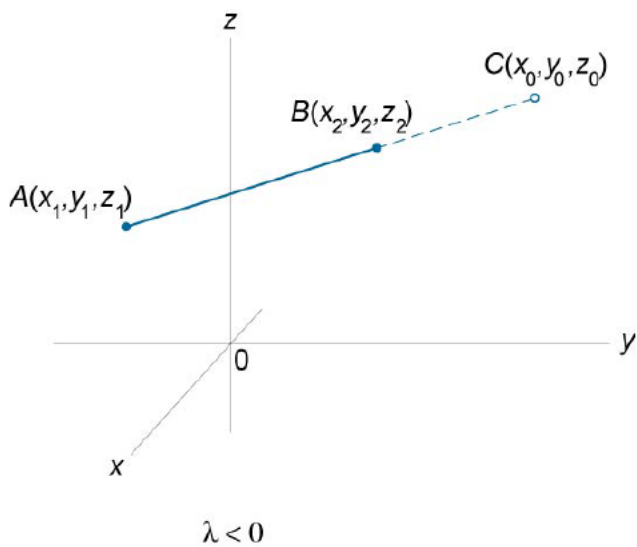


Figure 125.

672. Midpoint of a Line Segment

$$x_0 = \frac{x_1 + x_2}{2}, y_0 = \frac{y_1 + y_2}{2}, z_0 = \frac{z_1 + z_2}{2}, \lambda = 1.$$

673. Area of a Triangle

The area of a triangle with vertices $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, and $P_3(x_3, y_3, z_3)$ is given by

$$S = \frac{1}{2} \sqrt{\begin{vmatrix} y_1 & z_1 & 1 \\ y_2 & z_2 & 1 \\ y_3 & z_3 & 1 \end{vmatrix}^2 + \begin{vmatrix} z_1 & x_1 & 1 \\ z_2 & x_2 & 1 \\ z_3 & x_3 & 1 \end{vmatrix}^2 + \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2}.$$

674. Volume of a Tetrahedron

The volume of a tetrahedron with vertices $P_1(x_1, y_1, z_1)$, $P_2(x_2, y_2, z_2)$, $P_3(x_3, y_3, z_3)$, and $P_4(x_4, y_4, z_4)$ is given by

$$V = \pm \frac{1}{6} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix},$$

or

$$V = \pm \frac{1}{6} \begin{vmatrix} x_1 - x_4 & y_1 - y_4 & z_1 - z_4 \\ x_2 - x_4 & y_2 - y_4 & z_2 - z_4 \\ x_3 - x_4 & y_3 - y_4 & z_3 - z_4 \end{vmatrix}.$$

Note: We choose the sign (+) or (-) so that to get a positive answer for volume.

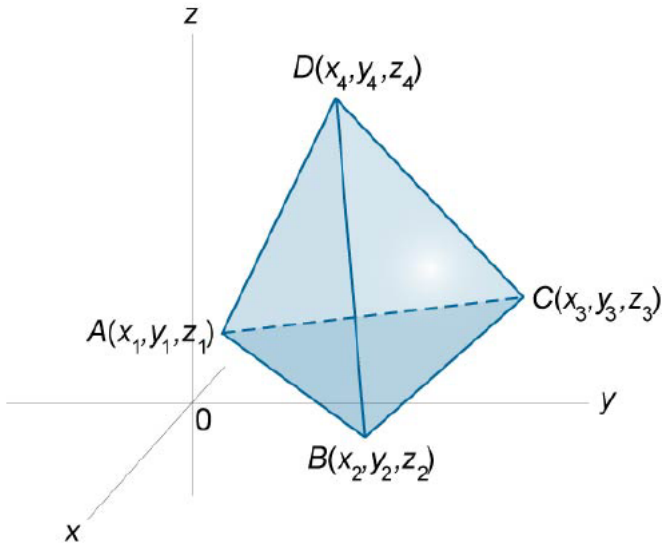


Figure 126.