Chapter 8
Differential Calculus

Functions: f, g, y, u, v
Argument (independent variable): x
Real numbers: a, b, c, d
Natural number: n
Angle: α
Inverse function: f^{-1}

8.1 Functions and Their Graphs

723. Even Function
$f(-x)=f(x)$

724. Odd Function
$f(-x)=-f(x)$

725. Periodic Function
$f(x+nT)=f(x)$

726. Inverse Function
$y=f(x)$ is any function, $x=g(y)$ or $y=f^{-1}(x)$ is its inverse function.

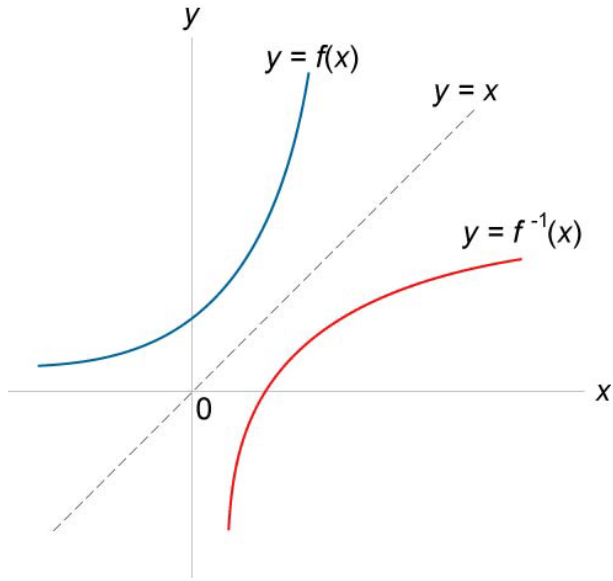

Figure 152.

727. Composite Function
$y = f(u),\ u = g(x),\ y = f(g(x))$ is a composite function.

728. Linear Function
$y = ax + b,\ \ x \in R,\ \ a = \tan\alpha$ is the slope of the line, b is the y-intercept.

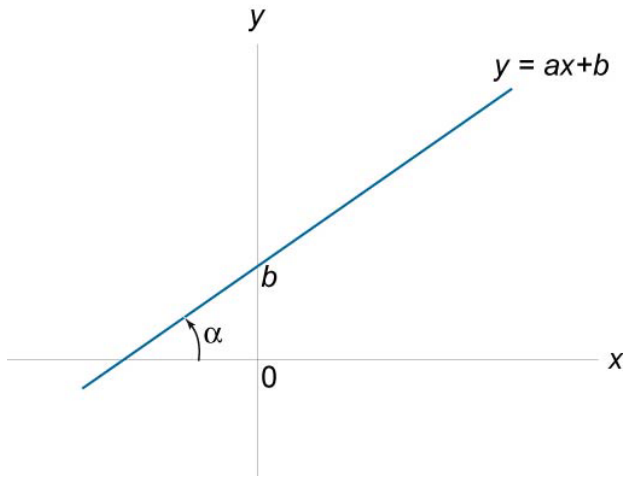

Figure 153.

729. Quadratic Function
$y = x^2,\ x \in R$.

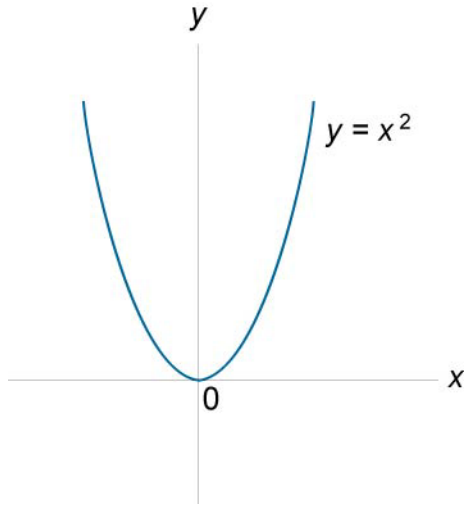

Figure 154.

730. $y = ax^2 + bx + c,\ x \in R$.

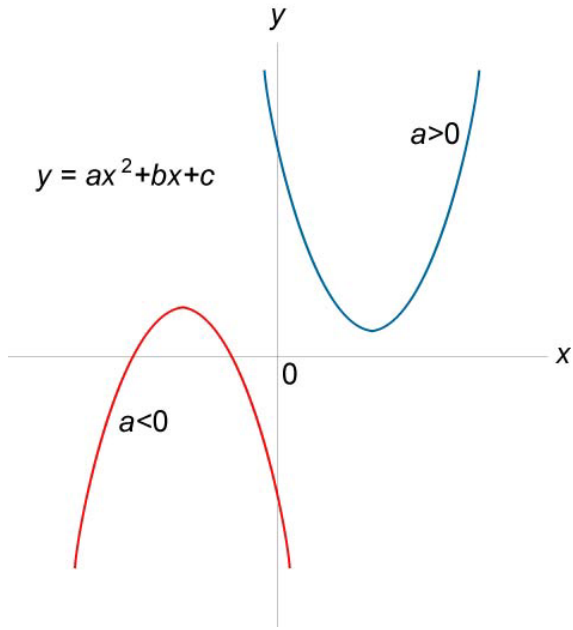

Figure 155.

731. Cubic Function

$y = x^3,\ x \in R$.

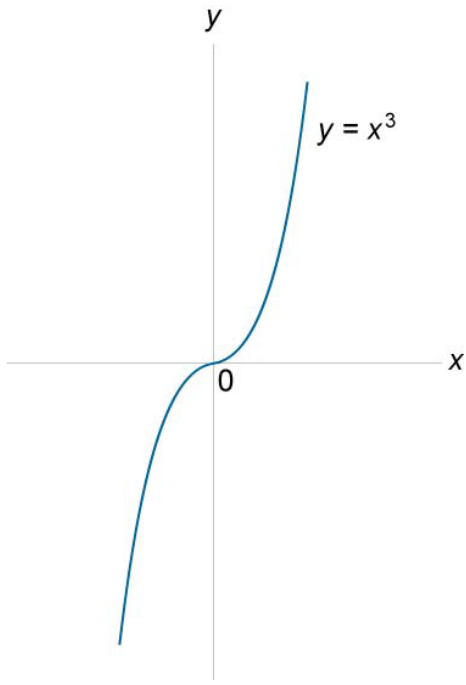

Figure 156.

732. $y = ax^3 + bx^2 + cx + d,\ x \in R.$

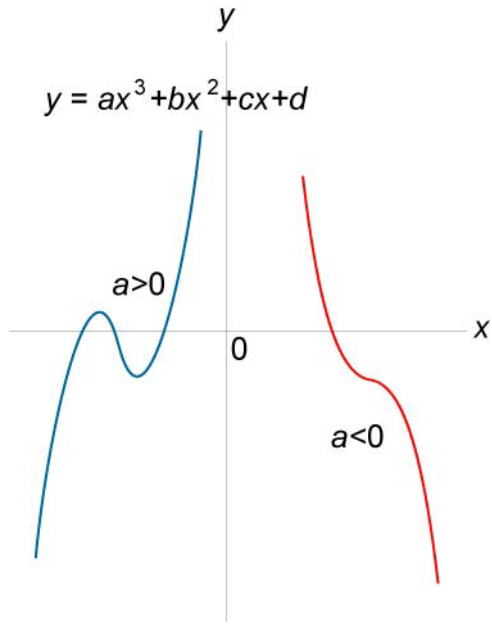

Figure 157.

733. Power Function
$y = x^n,\ n \in N.$

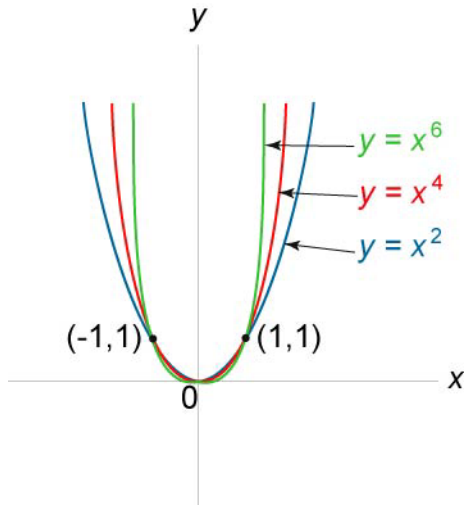

Figure 158.

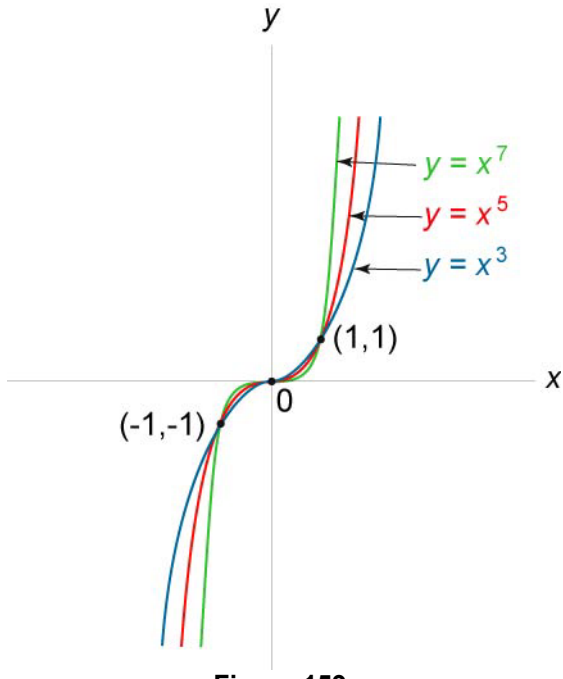

Figure 159.

734. Square Root Function

$y = \sqrt{x}\,,\ x \in [0, \infty)$.

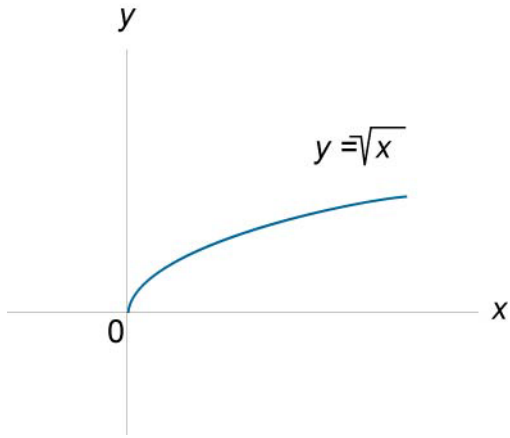

Figure 160.

735. Exponential Functions

$y = a^x,\ a > 0,\ a \neq 1,$

$y = e^x$ if $a = e,\ e = 2.71828182846\ldots$

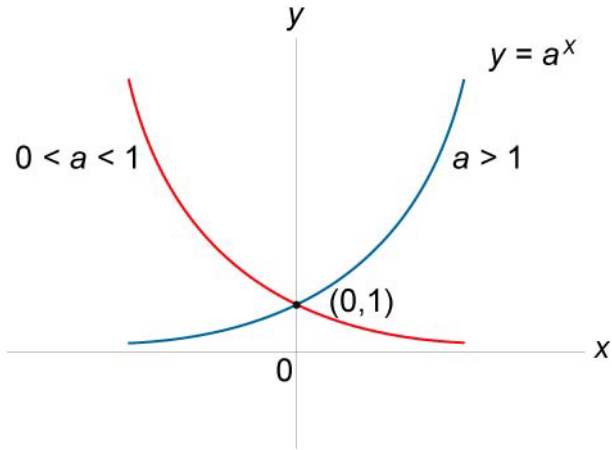

Figure 161.

736. Logarithmic Functions

$y = \log_a x$, $x \in (0, \infty)$, $a > 0$, $a \neq 1$,

$y = \ln x$ if $a = e$, $x > 0$.

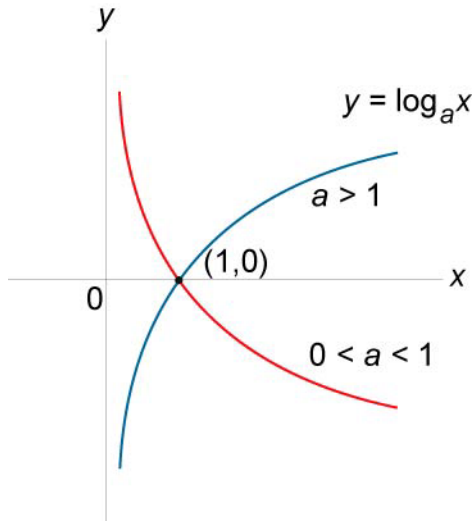

Figure 162.

737. Hyperbolic Sine Function

$$y = \sinh x, \ \sinh x = \frac{e^x - e^{-x}}{2}, \ x \in R.$$

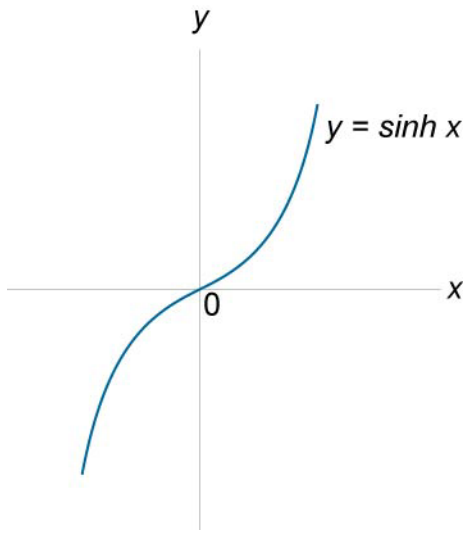

Figure 163.

738. Hyperbolic Cosine Function

$$y = \cosh x,\ \cosh x = \frac{e^x + e^{-x}}{2},\ x \in R.$$

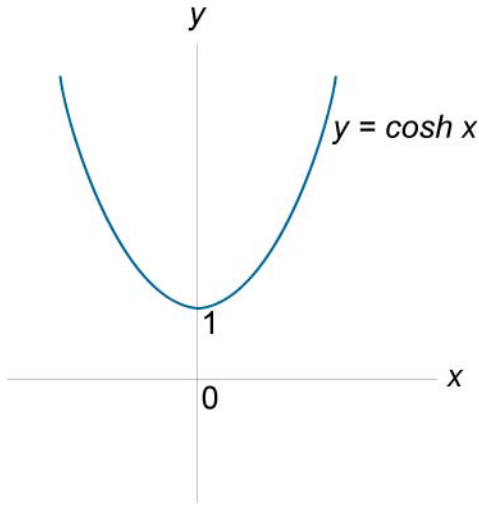

Figure 164.

739. Hyperbolic Tangent Function

$$y = \tanh x\,,\ y = \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}},\ x \in R\,.$$

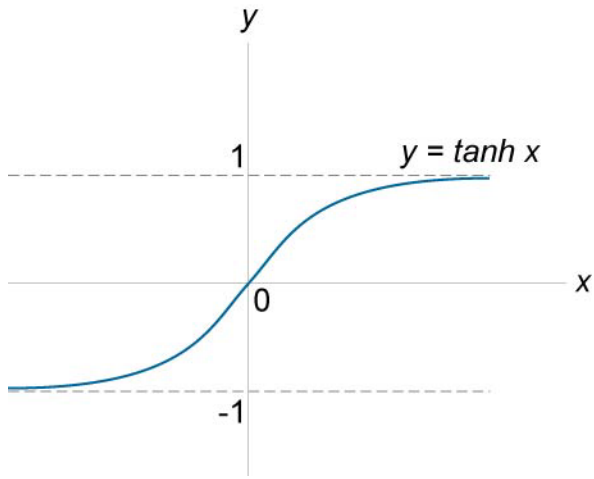

Figure 165.

740. Hyperbolic Cotangent Function

$$y = \coth x\,,\ y = \coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}},\ x \in R\,,\ x \neq 0\,.$$

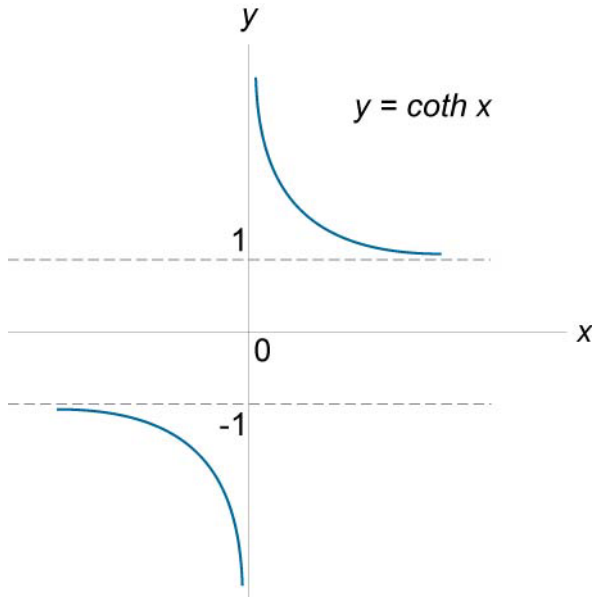

Figure 166.

741. Hyperbolic Secant Function

$$y = \operatorname{sech} x,\ y = \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}},\ x \in R.$$

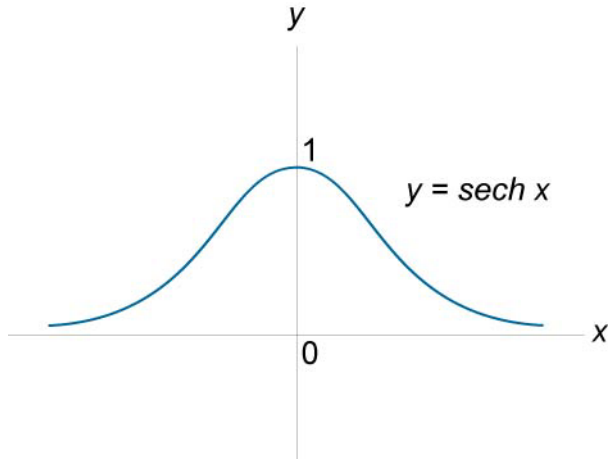

Figure 167.

742. Hyperbolic Cosecant Function

$$y = \operatorname{csch} x,\ y = \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}},\ x \in R,\ x \neq 0.$$

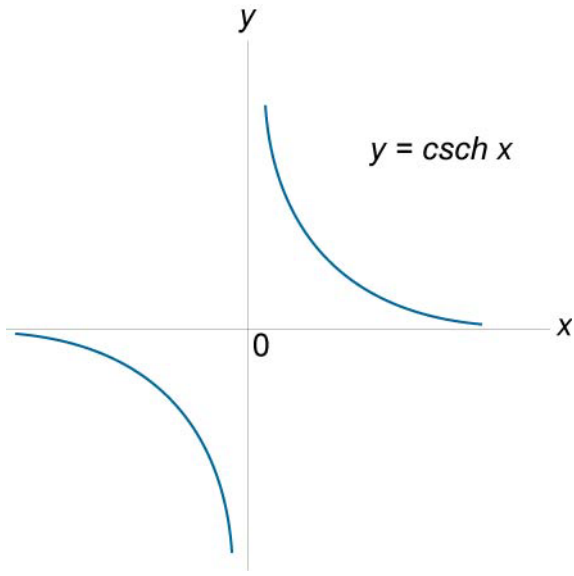

Figure 168.

743. Inverse Hyperbolic Sine Function

$$y = \operatorname{arcsinh} x,\ x \in R.$$

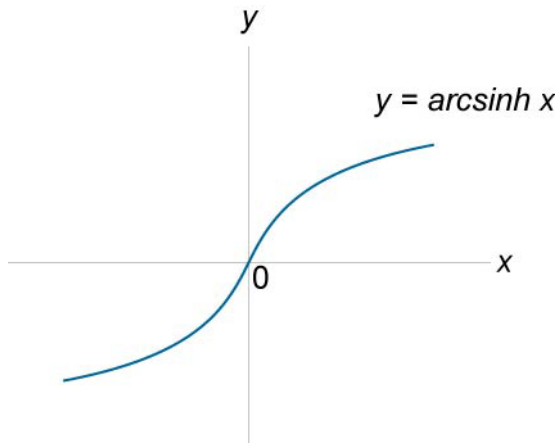

Figure 169.

744. Inverse Hyperbolic Cosine Function
$y = \operatorname{arccosh} x,\ x \in [1, \infty)$.

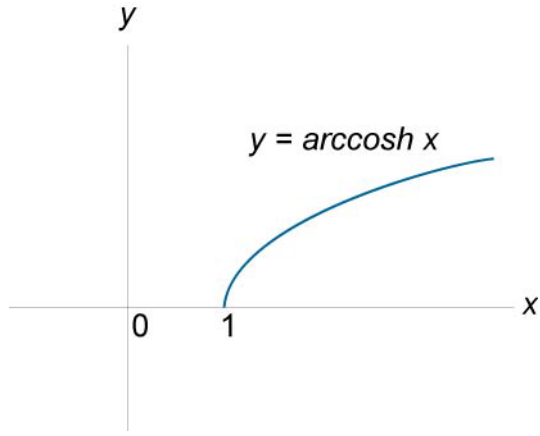

Figure 170.

745. Inverse Hyperbolic Tangent Function
$y = \operatorname{arctanh} x,\ x \in (-1, 1)$.

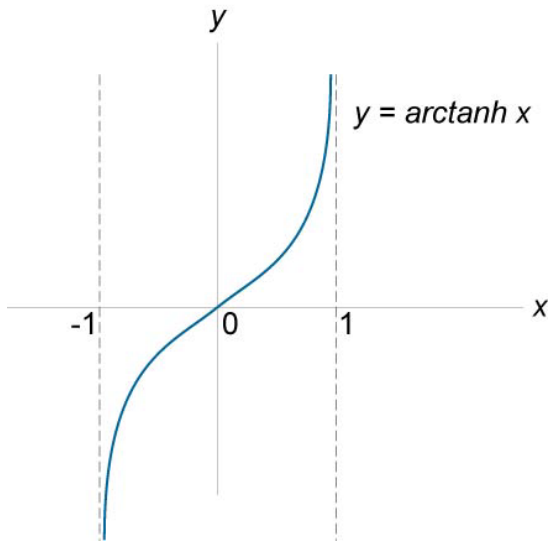

Figure 171.

746. Inverse Hyperbolic Cotangent Function
$y = \operatorname{arccoth} x,\ x \in (-\infty, -1) \cup (1, \infty).$

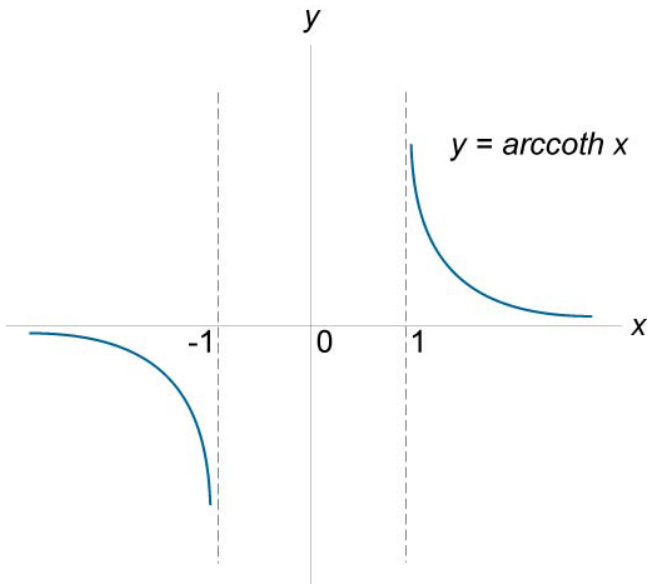

Figure 172.

747. Inverse Hyperbolic Secant Function
$y = \operatorname{arcsech} x,\ x \in (0, 1]$.

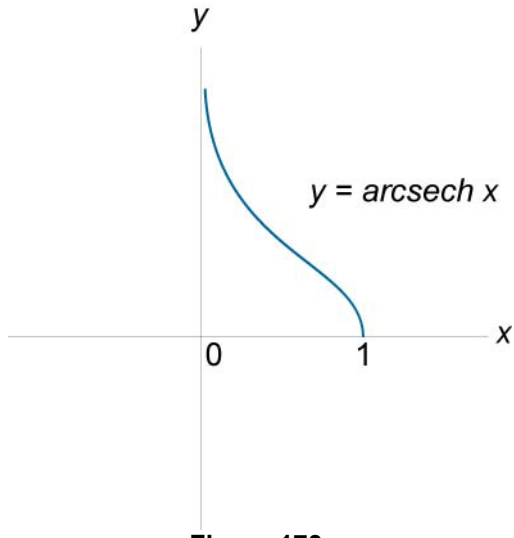

Figure 173.

748. Inverse Hyperbolic Cosecant Function
$y = \operatorname{arccsch} x,\ x \in R,\ x \neq 0.$

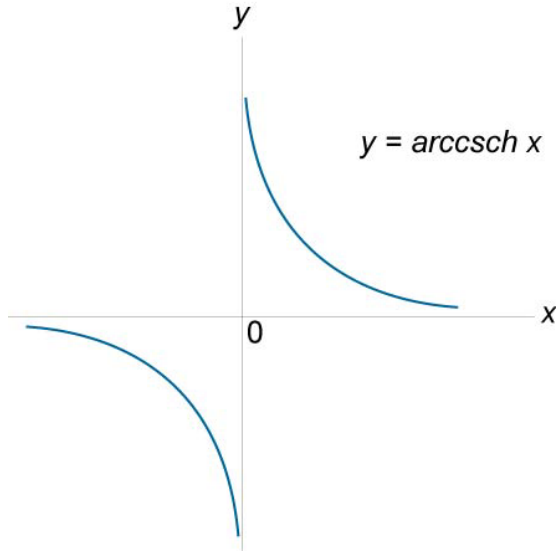

Figure 174.