

8.3 Definition and Properties of the Derivative

Functions: f, g, y, u, v
 Independent variable: x
 Real constant: k
 Angle: α

$$764. \quad y'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

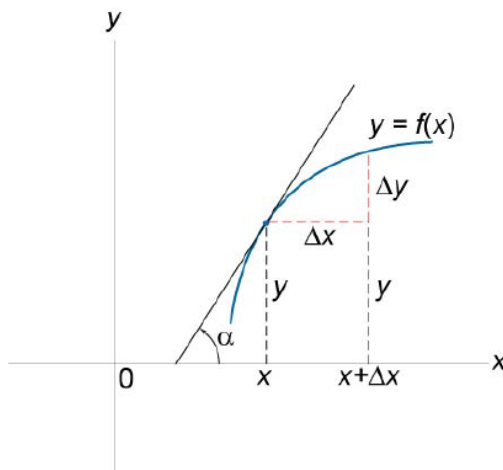


Figure 175.

$$765. \quad \frac{dy}{dx} = \tan \alpha$$

$$766. \quad \frac{d(u + v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$767. \quad \frac{d(u - v)}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$768. \quad \frac{d(ku)}{dx} = k \frac{du}{dx}$$

$$769. \quad \text{Product Rule} \\ \frac{d(u \cdot v)}{dx} = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

770. Quotient Rule

$$\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{\frac{du}{dx} \cdot v - u \cdot \frac{dv}{dx}}{v^2}$$

771. Chain Rule

$$y = f(g(x)), \quad u = g(x),$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}.$$

772. Derivative of Inverse Function

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}},$$

where $x(y)$ is the inverse function of $y(x)$.

773. Reciprocal Rule

$$\frac{d}{dx} \left(\frac{1}{y} \right) = -\frac{\frac{dy}{dx}}{y^2}$$

774. Logarithmic Differentiation

$$y = f(x), \quad \ln y = \ln f(x),$$

$$\frac{dy}{dx} = f(x) \cdot \frac{d}{dx} [\ln f(x)].$$