

9.3 Integrals of Irrational Functions

$$905. \int \frac{dx}{\sqrt{ax+b}} = \frac{2}{a} \sqrt{ax+b} + C$$

$$906. \int \sqrt{ax+b} dx = \frac{2}{3a} (ax+b)^{3/2} + C$$

$$907. \int \frac{x dx}{\sqrt{ax+b}} = \frac{2(ax-2b)}{3a^2} \sqrt{ax+b} + C$$

$$908. \int x \sqrt{ax+b} dx = \frac{2(3ax-2b)}{15a^2} (ax+b)^{3/2} + C$$

$$909. \int \frac{dx}{(x+c)\sqrt{ax+b}} = \frac{1}{\sqrt{b-ac}} \ln \left| \frac{\sqrt{ax+b} - \sqrt{b-ac}}{\sqrt{ax+b} + \sqrt{b-ac}} \right| + C,$$

$b-ac > 0.$

$$910. \int \frac{dx}{(x+c)\sqrt{ax+b}} = \frac{1}{\sqrt{ac-b}} \arctan \sqrt{\frac{ax+b}{ac-b}} + C,$$

$b-ac < 0.$

$$911. \int \sqrt{\frac{ax+b}{cx+d}} dx = \frac{1}{c} \sqrt{(ax+b)(cx+d)} -$$

$$- \frac{ad-bc}{c\sqrt{ac}} \ln \left| \sqrt{a(cx+d)} + \sqrt{c(ax+b)} \right| + C, a > 0.$$

$$912. \int \sqrt{\frac{ax+b}{cx+d}} dx = \frac{1}{c} \sqrt{(ax+b)(cx+d)} -$$

$$- \frac{ad-bc}{c\sqrt{ac}} \arctan \sqrt{\frac{a(cx+d)}{c(ax+b)}} + C, (a < 0, c > 0).$$

$$913. \int x^2 \sqrt{a+bx} dx = \frac{2(8a^2 - 12abx + 15b^2x^2)}{105b^3} \sqrt{(a+bx)^3} + C$$

$$914. \int \frac{x^2 dx}{\sqrt{a+bx}} = \frac{2(8a^2 - 4abx + 3b^2x^2)}{15b^3} \sqrt{a+bx} + C$$

$$915. \int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right| + C, a > 0.$$

$$916. \int \frac{dx}{x\sqrt{a+bx}} = \frac{2}{\sqrt{-a}} \arctan \left| \frac{a+bx}{-a} \right| + C, \quad a < 0.$$

$$917. \int \sqrt{\frac{a-x}{b+x}} dx = \sqrt{(a-x)(b+x)} + (a+b) \arcsin \sqrt{\frac{x+b}{a+b}} + C$$

$$918. \int \sqrt{\frac{a+x}{b-x}} dx = -\sqrt{(a+x)(b-x)} - (a+b) \arcsin \sqrt{\frac{b-x}{a+b}} + C$$

$$919. \int \sqrt{\frac{1+x}{1-x}} dx = -\sqrt{1-x^2} + \arcsin x + C$$

$$920. \int \frac{dx}{\sqrt{(x-a)(b-a)}} = 2 \arcsin \sqrt{\frac{x-a}{b-a}} + C$$

$$921. \int \sqrt{a+bx-cx^2} dx = \frac{2cx-b}{4c} \sqrt{a+bx-cx^2} + \frac{b^2-4ac}{8\sqrt{c^3}} \arcsin \frac{2cx-b}{\sqrt{b^2+4ac}} + C$$

$$922. \int \frac{dx}{\sqrt{ax^2+bx+c}} = \frac{1}{\sqrt{a}} \ln \left| 2ax+b+2\sqrt{a(ax^2+bx+c)} \right| + C, \\ a > 0.$$

$$923. \int \frac{dx}{\sqrt{ax^2+bx+c}} = -\frac{1}{\sqrt{a}} \arcsin \frac{2ax+b}{4a} \sqrt{b^2-4ac} + C, \quad a < 0.$$

$$924. \int \sqrt{x^2+a^2} dx = \frac{x}{2} \sqrt{x^2+a^2} + \frac{a^2}{2} \ln \left| x + \sqrt{x^2+a^2} \right| + C$$

$$925. \int x\sqrt{x^2+a^2} dx = \frac{1}{3}(x^2+a^2)^{3/2} + C$$

$$926. \int x^2\sqrt{x^2+a^2} dx = \frac{x}{8}(2x^2+a^2)\sqrt{x^2+a^2} - \frac{a^4}{8}\ln|x+\sqrt{x^2+a^2}| + C$$

$$927. \int \frac{\sqrt{x^2+a^2}}{x^2} dx = -\frac{\sqrt{x^2+a^2}}{x} + \ln|x+\sqrt{x^2+a^2}| + C$$

$$928. \int \frac{dx}{\sqrt{x^2+a^2}} = \ln|x+\sqrt{x^2+a^2}| + C$$

$$929. \int \frac{\sqrt{x^2+a^2}}{x} dx = \sqrt{x^2+a^2} + a \ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$$

$$930. \int \frac{xdx}{\sqrt{x^2+a^2}} = \sqrt{x^2+a^2} + C$$

$$931. \int \frac{x^2 dx}{\sqrt{x^2+a^2}} = \frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\ln|x+\sqrt{x^2+a^2}| + C$$

$$932. \int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{1}{a}\ln\left|\frac{x}{a+\sqrt{x^2+a^2}}\right| + C$$

$$933. \int \sqrt{x^2-a^2} dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln|x+\sqrt{x^2-a^2}| + C$$

$$934. \int x\sqrt{x^2-a^2} dx = \frac{1}{3}(x^2-a^2)^{3/2} + C$$

$$935. \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} + a \arcsin \frac{a}{x} + C$$

$$936. \int \frac{\sqrt{x^2 - a^2}}{x^2} dx = -\frac{\sqrt{x^2 - a^2}}{x} + \ln|x + \sqrt{x^2 - a^2}| + C$$

$$937. \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$938. \int \frac{x dx}{\sqrt{x^2 - a^2}} = \sqrt{x^2 - a^2} + C$$

$$939. \int \frac{x^2 dx}{\sqrt{x^2 - a^2}} = \frac{x}{2} \sqrt{x^2 - a^2} + \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$940. \int \frac{dx}{x\sqrt{x^2 - a^2}} = -\frac{1}{a} \arcsin \frac{a}{x} + C$$

$$941. \int \frac{dx}{(x+a)\sqrt{x^2 - a^2}} = \frac{1}{a} \sqrt{\frac{x-a}{x+a}} + C$$

$$942. \int \frac{dx}{(x-a)\sqrt{x^2 - a^2}} = -\frac{1}{a} \sqrt{\frac{x+a}{x-a}} + C$$

$$943. \int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$$

$$944. \int \frac{dx}{(x^2 - a^2)^{3/2}} = -\frac{x}{a^2 \sqrt{x^2 - a^2}} + C$$

$$945. \int (x^2 - a^2)^{3/2} dx = -\frac{x}{8}(2x^2 - 5a^2)\sqrt{x^2 - a^2} + \frac{3a^4}{8} \ln|x + \sqrt{x^2 - a^2}| + C$$

$$946. \int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$947. \int x\sqrt{a^2 - x^2} dx = -\frac{1}{3}(a^2 - x^2)^{3/2} + C$$

$$948. \int x^2\sqrt{a^2 - x^2} dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a} + C$$

$$949. \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} + a \ln \left| \frac{x}{a + \sqrt{a^2 - x^2}} \right| + C$$

$$950. \int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin \frac{x}{a} + C$$

$$951. \int \frac{dx}{\sqrt{1 - x^2}} = \arcsin x + C$$

$$952. \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$953. \int \frac{x dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2} + C$$

$$954. \int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

$$955. \int \frac{dx}{(x+a)\sqrt{a^2-x^2}} = -\frac{1}{2} \sqrt{\frac{a-x}{a+x}} + C$$

$$956. \int \frac{dx}{(x-a)\sqrt{a^2-x^2}} = -\frac{1}{2} \sqrt{\frac{a+x}{a-x}} + C$$

$$957. \int \frac{dx}{(x+b)\sqrt{a^2-x^2}} = \frac{1}{\sqrt{b^2-a^2}} \arcsin \frac{bx+a^2}{a(x+b)} + C, \quad b > a.$$

$$958. \int \frac{dx}{(x+b)\sqrt{a^2-x^2}} = \frac{1}{\sqrt{a^2-b^2}} \ln \left| \frac{x+b}{\sqrt{a^2-b^2}\sqrt{a^2-x^2} + a^2+bx} \right| + C,$$

$b < a.$

$$959. \int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$$

$$960. \int (a^2-x^2)^{3/2} dx = \frac{x}{8}(5a^2-2x^2)\sqrt{a^2-x^2} + \frac{3a^4}{8} \arcsin \frac{x}{a} + C$$

$$961. \int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}} + C$$