Chapter 5

Matrices and Determinants

Matrices: A, B, C

Elements of a matrix: a_i, b_i, a_{ij}, b_{ij}, c_{ij}

Determinant of a matrix: det A

Minor of an element a_{ij}: M_{ij}

Cofactor of an element a_{ij}: C_{ij}

Transpose of a matrix: A^T, $\tilde{A}$

Adjoint of a matrix: adj A

Trace of a matrix: tr A

Inverse of a matrix: A^{-1}

Real number: k

Real variables: x_i

Natural numbers: m, n

5.1 Determinants

513. Second Order Determinant

$$\det A = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$$

514. Third Order Determinant

$$\det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$

$$- a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - a_{13}a_{22}a_{31}$$
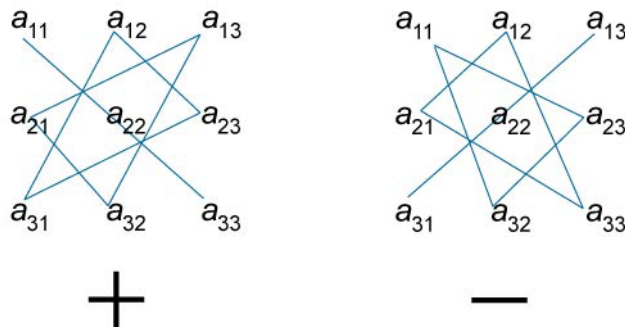
515. Sarrus Rule (Arrow Rule)

Figure 72.

516. N-th Order Determinant

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nj} & \cdots & a_{nn} \end{vmatrix}$$

517. Minor

The minor M_{ij} associated with the element a_{ij} of n-th order matrix A is the $(n-1)$-th order determinant derived from the matrix A by deletion of its i-th row and j-th column.

518. Cofactor

$$C_{ij} = (-1)^{i+j} M_{ij}$$

519. Laplace Expansion of n-th Order Determinant

Laplace expansion by elements of the i-th row

$$\det A = \sum_{j=1}^{n} a_{ij} C_{ij} \,, \ i = 1, 2, \ldots, n.$$

Laplace expansion by elements of the j-th column

$$\det A = \sum_{i=1}^{n} a_{ij} C_{ij} \,, \ j = 1, 2, \ldots, n.$$