

1.4 Complex Numbers

Natural number: n

Imaginary unit: i

Complex number: z

Real part: a, c

Imaginary part: bi, di

Modulus of a complex number: r, r_1, r_2

Argument of a complex number: $\varphi, \varphi_1, \varphi_2$

46.

$i^1 = i$	$i^5 = i$	$i^{4n+1} = i$
$i^2 = -1$	$i^6 = -1$	$i^{4n+2} = -1$
$i^3 = -i$	$i^7 = -i$	$i^{4n+3} = -i$
$i^4 = 1$	$i^8 = 1$	$i^{4n} = 1$

47. $z = a + bi$

48. Complex Plane

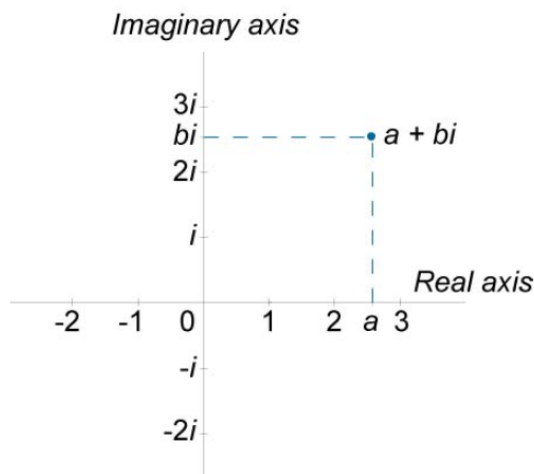


Figure 6.

$$49. (a + bi) + (c + di) = (a + c) + (b + d)i$$

$$50. (a + bi) - (c + di) = (a - c) + (b - d)i$$

$$51. (a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$52. \frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2} \cdot i$$

53. Conjugate Complex Numbers

$$\overline{a + bi} = a - bi$$

$$54. a = r \cos \phi, b = r \sin \phi$$

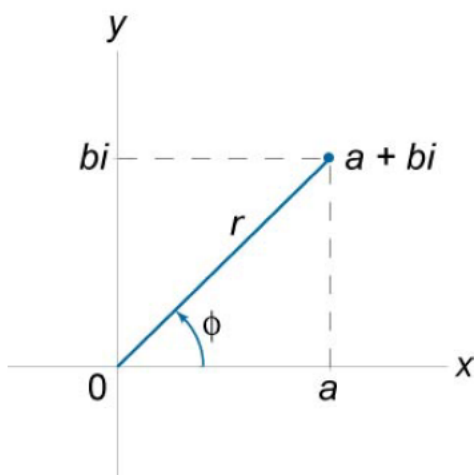


Figure 7.

55. Polar Presentation of Complex Numbers

$$a + bi = r(\cos \varphi + i \sin \varphi)$$

56. Modulus and Argument of a Complex Number

If $a + bi$ is a complex number, then

$$r = \sqrt{a^2 + b^2} \quad (\text{modulus}),$$

$$\varphi = \arctan \frac{b}{a} \quad (\text{argument}).$$

57. Product in Polar Representation

$$\begin{aligned} z_1 \cdot z_2 &= r_1(\cos \varphi_1 + i \sin \varphi_1) \cdot r_2(\cos \varphi_2 + i \sin \varphi_2) \\ &= r_1 r_2 [\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2)] \end{aligned}$$

58. Conjugate Numbers in Polar Representation

$$\overline{r(\cos \varphi + i \sin \varphi)} = r[\cos(-\varphi) + i \sin(-\varphi)]$$

59. Inverse of a Complex Number in Polar Representation

$$\frac{1}{r(\cos \varphi + i \sin \varphi)} = \frac{1}{r} [\cos(-\varphi) + i \sin(-\varphi)]$$

60. Quotient in Polar Representation

$$\frac{z_1}{z_2} = \frac{r_1(\cos \varphi_1 + i \sin \varphi_1)}{r_2(\cos \varphi_2 + i \sin \varphi_2)} = \frac{r_1}{r_2} [\cos(\varphi_1 - \varphi_2) + i \sin(\varphi_1 - \varphi_2)]$$

61. Power of a Complex Number

$$z^n = [r(\cos \varphi + i \sin \varphi)]^n = r^n [\cos(n\varphi) + i \sin(n\varphi)]$$

62. Formula “De Moivre”

$$(\cos \varphi + i \sin \varphi)^n = \cos(n\varphi) + i \sin(n\varphi)$$

63. Nth Root of a Complex Number

$$\sqrt[n]{z} = \sqrt[n]{r(\cos \varphi + i \sin \varphi)} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2\pi k}{n} + i \sin \frac{\varphi + 2\pi k}{n} \right),$$

where

$$k = 0, 1, 2, \dots, n-1.$$

64. Euler’s Formula

$$e^{ix} = \cos x + i \sin x$$