6.5 Scalar Product

573. Scalar Product of Vectors $\vec{u}$ and $\vec{v}$
$\vec{u}\cdot\vec{v}=|\vec{u}|\cdot|\vec{v}|\cdot\cos\theta$,
where θ is the angle between vectors $\vec{u}$ and $\vec{v}$.

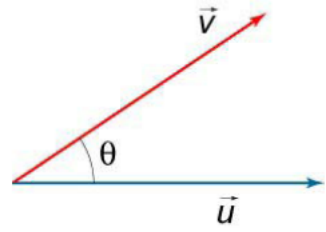

Figure 82.

574. Scalar Product in Coordinate Form
If $\vec{u}=(X_1,Y_1,Z_1)$, $\vec{v}=(X_2,Y_2,Z_2)$, then
$\vec{u}\cdot\vec{v}=X_1X_2+Y_1Y_2+Z_1Z_2$.

575. Angle Between Two Vectors
If $\vec{u}=(X_1,Y_1,Z_1)$, $\vec{v}=(X_2,Y_2,Z_2)$, then
$$\cos\theta=\frac{X_1X_2+Y_1Y_2+Z_1Z_2}{\sqrt{X_1^{\,2}+Y_1^{\,2}+Z_1^{\,2}}\sqrt{X_2^{\,2}+Y_2^{\,2}+Z_2^{\,2}}}.$$

576. Commutative Property
$\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$

577. Associative Property
$(\lambda\vec{u})\cdot(\mu\vec{v})=\lambda\mu\vec{u}\cdot\vec{v}$

578. Distributive Property
$\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$

579. $\vec{u}\cdot\vec{v}=0$ if $\vec{u},\vec{v}$ are orthogonal ($\theta=\frac{\pi}{2}$).

580. $\vec{u}\cdot\vec{v}>0$ if $0<\theta<\frac{\pi}{2}$.

581. $\vec{u}\cdot\vec{v}<0$ if $\frac{\pi}{2}<\theta<\pi$.

582. $\vec{u}\cdot\vec{v}\le|\vec{u}|\cdot|\vec{v}|$

583. $\vec{u}\cdot\vec{v}=|\vec{u}|\cdot|\vec{v}|$ if $\vec{u},\vec{v}$ are parallel ($\theta=0$).

584. If $\vec{u}=(X_1,Y_1,Z_1)$, then
$\vec{u}\cdot\vec{u}=\vec{u}^2=|\vec{u}|^2=X_1^{\,2}+Y_1^{\,2}+Z_1^{\,2}$.

585. $\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}=1$

586. $\vec{i}\cdot\vec{j}=\vec{j}\cdot\vec{k}=\vec{k}\cdot\vec{i}=0$