6.7 Triple Product

598. Scalar Triple Product

$$[\vec{u}\vec{v}\vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w}) = \vec{v}\cdot(\vec{w}\times\vec{u}) = \vec{w}\cdot(\vec{u}\times\vec{v})$$

599. $[\vec{u}\vec{v}\vec{w}] = [\vec{w}\vec{u}\vec{v}] = [\vec{v}\vec{w}\vec{u}] = -[\vec{v}\vec{u}\vec{w}] = -[\vec{w}\vec{v}\vec{u}] = -[\vec{u}\vec{w}\vec{v}]$

600. $k\vec{u}\cdot(\vec{v}\times\vec{w}) = k[\vec{u}\vec{v}\vec{w}]$

601. Scalar Triple Product in Coordinate Form

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \\ X_3 & Y_3 & Z_3 \end{vmatrix},$$

where

$\vec{u} = (X_1, Y_1, Z_1)$, $\vec{v} = (X_2, Y_2, Z_2)$, $\vec{w} = (X_3, Y_3, Z_3)$.

602. Volume of Parallelepiped

$$V = |\vec{u}\cdot(\vec{v}\times\vec{w})|$$

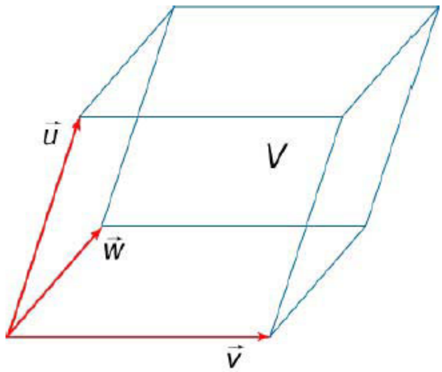

Figure 84.

603. Volume of Pyramid

$$V = \frac{1}{6}\left|\vec{u}\cdot(\vec{v}\times\vec{w})\right|$$

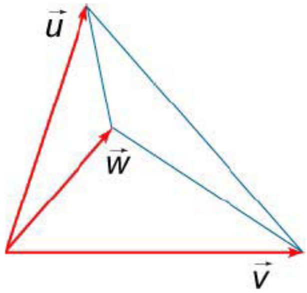

Figure 85.

604. If $\vec{u}\cdot(\vec{v}\times\vec{w})=0$, then the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are linearly dependent, so $\vec{w}=\lambda\vec{u}+\mu\vec{v}$ for some scalars λ and μ.

605. If $\vec{u}\cdot(\vec{v}\times\vec{w})\neq 0$, then the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ are linearly independent.

606. Vector Triple Product

$$\vec{u}\times(\vec{v}\times\vec{w})=(\vec{u}\cdot\vec{w})\vec{v}-(\vec{u}\cdot\vec{v})\vec{w}$$