(d) Laplace's and Poisson Equations

Since $\vec{E} = -\vec{\nabla} V$ and $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ $\Rightarrow \nabla^2 V = -\frac{\rho}{\varepsilon_0}$

This is known as **Poisson's equation**.

In regions where there is no charge, so that $\rho = 0$, Poisson's equation reduces to Laplace's equation,

$$\nabla^2 V = 0.$$

Example: Potential in a region of space is given by, $\phi = \phi_0 e^{-ax^2}$ where ϕ_0 and a is constant. Then find the charge density in this region.

Solution: $\nabla^2 \phi = -\frac{\rho}{\varepsilon_0} \Rightarrow \rho = -\varepsilon_0 \left(\nabla^2 \phi\right) = -\varepsilon_0 \frac{\partial}{\partial x}\left[\phi_0 e^{-ax^2} \times -2ax\right]$

$$\Rightarrow \rho = 2a\phi_0\varepsilon_0 \frac{\partial}{\partial x}\left[xe^{-ax^2}\right] = 2a\phi_0\varepsilon_0\left[e^{-ax^2} + xe^{-ax^2}\left(-2ax\right)\right]$$

$$\Rightarrow \rho = 2a\phi_0\varepsilon_0 e^{-ax^2}\left[1-2ax^2\right]$$