(f) Basic Properties of Conductors and Capacitors

Basic properties of conductors are:

1. $\vec{E} = 0$ inside a conductor.
2. $\rho = 0$ inside a conductor.
3. Any net charge resides on the surface.
4. A conductor is an equipotential.
5. $\vec{E}$ is perpendicular to the surface, just outside a conductor.

Capacitors

Suppose we have two conductors, and we put charge $+Q$ on one and $-Q$ on the other. Since potential is constant over a conductor, we can write potential difference between them is:

$$V = V_+ - V_- = -\int_{(-)}^{(+)} \vec{E}.d\vec{l}$$

Since $\vec{E}$ is proportional to Q, so also is V. The constant of proportionality is called the capacitance of the arrangement:

$$C = \frac{Q}{V}.$$

Capacitance is a purely geometrical quantity, determined by the sizes, shapes, and separation of the two conductors.

To "charge up" a capacitor, we have to remove electrons from the positive plate and carry them to the negative plate. In doing so we have to fight against the electric field, which is pulling them back toward the positive conductor and pushing them away from the negative one. We have to find work done to charge the capacitor up to a final amount Q.

Suppose that at some intermediate stage in the process the charge on the positive plate is q, so that the potential difference is $\frac{q}{C}$. Thus work we must do to transport the next piece of charge, dq, is

$$dW = \left(\frac{q}{C}\right)dq$$

The total work necessary, then, to go from $q = 0$ to $q = Q$, is

$$W = \int_0^Q \left(\frac{q}{C}\right)dq = \frac{Q^2}{2C} \qquad \Rightarrow W = \frac{1}{2}CV^2 \qquad \because Q = CV$$