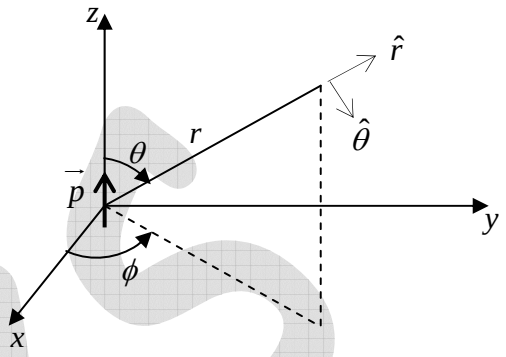
(g) Electric Dipoles

A electric dipole consists of two equal and opposite charges $(\pm q)$ separated by a distance d. Its dipole moment is $\vec{p} = q\vec{d}$ and its direction is from $-q$ to $+q$ charge.

The Electric Potential and Field of a Dipole

If we choose coordinates so that $\vec{p}$ (dipole moment) lies at the origin and points in the z-direction, then potential at (r,θ) is:



$$V_{dip}(r,\theta) = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0 r^2}\frac{\vec{p}.\hat{r}}{r^2}.$$

$\because \vec{p}.\hat{r} = (p\hat{z}).\hat{r} = p\cos\theta$

Since $\vec{E} = -\vec{\nabla}V$

$$\Rightarrow E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\varepsilon_o r^3}, E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = \frac{p\sin\theta}{4\pi\varepsilon_o r^3}, E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi} = 0.$$

The electric field of a dipole:

$$\vec{E}_{dip}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\hat{\theta}\right).$$

We can express $\vec{p} = \left(\vec{p}.\hat{r}\right)\hat{r} + \left(\vec{p}.\hat{\theta}\right)\hat{\theta} = p\cos\theta\hat{r} - p\sin\theta\hat{\theta}$.

Thus, $3\left(\vec{p}\cdot\hat{r}\right)\hat{r} - \vec{p} = 2p\cos\theta\,\hat{r} + p\sin\theta\hat{\theta}$

$$\Rightarrow \vec{E}_{dip}(r,\theta) = \frac{1}{4\pi\varepsilon_0 r^3}\left[3\left(\vec{p}\cdot\hat{r}\right)\hat{r} - \vec{p}\right]$$

Note:

(a) When a dipole is placed in a uniform electric field ($\vec{E}$), net force on the dipole is zero and it experiences a torque $\vec{\tau} = \vec{p}\times\vec{E}$ where $\vec{p} = q\vec{d}$.

(b) In non-uniform field, dipoles have net force $\vec{F} = \left(\vec{p}\cdot\vec{\nabla}\right)\vec{E}$ and torque $\vec{\tau} = \vec{p}\times\vec{E}$.

(c) Energy of an ideal dipole $\vec{p}$ in an electric field $\vec{E}$ is $U = -\vec{p}.\vec{E}$.

(d) Interaction energy of two dipoles separated by a distance $\vec{r}$ is

$$U = \frac{1}{4\pi\varepsilon_0 r^3}\left[\vec{p}_1\cdot\vec{p}_2 - 3\left(\vec{p}_1\cdot\hat{r}\right)\left(\vec{p}_2\cdot\hat{r}\right)\right]$$