

(h) Polarization and Bound Charges

When a piece of dielectric material is placed in an external field and if the substance consists of neutral atoms, the field will induce in a tiny dipole moment, pointing in the same direction as the field. If the material is made up of polar molecules each permanent dipole will experience a torque, tending to line it up along the field direction. (Random thermal motions compete with this process, so the alignment is never complete, especially at higher temperatures, and disappears almost at once when the field is removed.)

(Polarization) $\vec{P} \equiv$ dipole moment per unit volume

The Field of a Polarized Object (Bound Charges)

Suppose we have a piece of polarized material with polarization vector \vec{P} containing a lot of microscopic dipoles lined up.

For a single dipole of dipole moment \vec{p} we have

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{R}}{R^2} \text{ where } \vec{R} \text{ is the vector from the dipole to}$$

the point at which we are evaluating the potential.

$$\text{Thus, } V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\hat{R} \cdot \vec{P}(r')}{R^2} d\tau'; \text{ since } \vec{p} = \vec{P}(r') d\tau'$$

$$\text{By solving the above equation, we get } V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \oint_S \frac{1}{R} \vec{P} \cdot d\vec{a}' - \frac{1}{4\pi\epsilon_0} \int_V \left(\frac{1}{R} \right) (\vec{\nabla}' \cdot \vec{P}) d\tau'$$

The first term looks like the potential of a **surface bound charge**

$$\sigma_b = \vec{P} \cdot \hat{n} \quad (\text{where } \hat{n} \text{ is the normal unit vector})$$

The second term looks like the potential of a **volume bound charge**

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

Thus potential (and hence also the field) of a polarized object is the same as that produced by a volume charge density $\rho_b = -\vec{\nabla} \cdot \vec{P}$ plus a surface charge density $\sigma_b = \vec{P} \cdot \hat{n}$.

