

(i) Gauss Law in Presence of Dielectrics (The Electric Displacement)

Within the dielectric, the total charge density can be written as $\rho = \rho_b + \rho_f$ where ρ_b is volume bound charge ρ_f **free charge** density.

$$\text{From Gauss Law; } \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \epsilon_0 (\vec{\nabla} \cdot \vec{E}) = (\rho_b + \rho_f) = -\vec{\nabla} \cdot \vec{P} + \rho_f$$

where \vec{E} is now the **total field**, not just that portion generated by polarization.

$$\Rightarrow \vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$$

$$\Rightarrow \vec{\nabla} \cdot \vec{D} = \rho_f \quad \text{where } \vec{D} = \epsilon_0 \vec{E} + \vec{P} \text{ is known as the } \mathbf{electric\ displacement.}$$

Thus Gauss' law reads, $\vec{\nabla} \cdot \vec{D} = \rho_f$ or, in integral form $\oint \vec{D} \cdot d\vec{a} = Q_{f_{enc}}$, where $Q_{f_{enc}}$ denotes the total free charge enclosed in the volume.