(a) Basic Concepts, Matrix Addition, Scalar Multiplication

Basic Concepts

A **matrix** (plural **matrices**) is a rectangular array of numbers, symbols, or expressions, arranged in *rows* and *columns*. The individual items in a matrix are called its *elements* or *entries*. Thus a $m\times n$ matrix is of the form

$$A=\left[a_{ij}\right]=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & a_{ij} & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

In matrix, A has m rows and n column. a_{ij} is the element of i^{th} row and j^{th} column.

If number of row and number of column in matrix is equal $(m=n)$, then matrix is said to be **square matrix**. Then its diagonal containing the entries $a_{11},a_{22},....,a_{nn}$ is called main diagonal or principal diagonal of A.

A matrix that is not diagonal is called a **rectangular matrix**.

Transposition

A transpose A^T of a $m\times n$ matrix $A=\left[a_{ij}\right]$ is the $n\times m$ matrix that has the first row of A as its first column, the second row of A as its second column, and so on. Thus, the transpose of A is

$$A^T=\left[a_{ji}\right]=\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}$$

Symmetric matrices and skew-symmetric matrices

Matrices whose transpose equals the matrix are called symmetric matrix. Matrices whose transpose equals the minus of matrix are called anti-symmetric matrix

$A^T=A$ (symmetric matrix) and $A^T=-A$ (anti-symmetric matrix)

Equality of Matrices

Two matrices $A=\left[a_{ij}\right]$ and $B=\left[b_{ij}\right]$ are equal, written $A=B$, if and only if they have the same size and the corresponding entries are equal, that is $a_{11}=b_{11},a_{12}=b_{12}$ and so on.

Matrix Addition

Addition is defined only for matrices $A=\left[a_{ij}\right]$ and $B=\left[b_{ij}\right]$ of the same size; their sum written $A+B$, is then obtained by adding the corresponding entries. Matrices of different sizes cannot be added

If $A=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$ and $B=\begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix}$

Then $A+B=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \cdots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \cdots & a_{2n}+b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \cdots & a_{mn}+b_{mn} \end{bmatrix}$

Scalar Multiplication

The product of any $m\times n$ matrix $A=\left[a_{ij}\right]$ and any scalar λ written λA is the $m\times n$ matrix $\lambda A=\left[\lambda a_{ij}\right]$ obtained by multiplying each entry in A by λ. Thus

$$\lambda A=\begin{bmatrix} \lambda a_{11} & \lambda a_{12} & \cdots & \lambda a_{1n} \\ \lambda a_{21} & \lambda a_{22} & \cdots & \lambda a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda a_{m1} & \lambda a_{m2} & \cdots & \lambda a_{mn} \end{bmatrix}$$

Here $(-1)A$ is simply written $-A$ and is called the negative of A. Similarly, $(-k)A$ is simply written $-kA$. Also, $A+(-B)$ is written $A-B$ and is called the difference of A and B (which must have the same size).

For matrices of the same size $m\times n$, we obtain for addition

(i) $A+B=B+A$

(ii) $(A+B)+C=A+(B+C)$

(iii) $A+0=A$

(iv) $A+(-A)=0$ here 0 denote the zero matrix of size $m\times n$.

And for scalar multiplication

(i) $\lambda(A+B)=\lambda A+\lambda B$

(ii) $(\lambda+k)A=\lambda A+kA$

(iii) $\lambda(kA)=(\lambda k)A$

(iv) $1A=A$

Transposition of sum can be done term by term $(A+B)^T=A^T+B^T$

And for scalar multiplication we have $(\lambda A)^T=\lambda A^T$