(b) Matrix Multiplication

The product $C = AB$ (in this order) of an $m \times n$ matrix $A = \left[a_{ij}\right]$ and $r \times p$ matrix $B = \left[b_{ij}\right]$ is defined if and only if $r = n$, that is

Number of rows of 2nd factor B= Number of columns of 1st factor A

and is then defined as the $m \times p$ matrix $C = \left[c_{ij}\right]$ with entries

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj} = a_{i1} b_{1j} + a_{i2} b_{2j} + ... + a_{in} b_{nj} \text{ where } i = 1, ..., m \text{ and } j = 1, ..., p.$$

Example: $AB = \begin{bmatrix} 4 & 3 \\ 7 & 2 \\ 9 & 0 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 4.2+3.1 & 4.5+3.6 \\ 7.2+2.1 & 7.5+2.6 \\ 9.2+0.1 & 9.5+0.6 \end{bmatrix} = \begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 18 & 45 \end{bmatrix}$

Properties of Matrix Multiplication

(i) $AB \neq BA$ in general

(ii) $AB = 0$ does not necessarily imply $A = 0$ or $B = 0$ or $BA = 0$.

(iii) $AC = AD$ does not necessarily imply $C = D$.

(iv) $(kA)B = k(AB) = A(kB)$

(v) $A(BC) = (AB)C$

(vi) $(A+B)C = AC + BC$

(vii) $(AB)^T = B^T A^T$