(c) Special Matrices

Triangular Matrices

Upper triangular matrices are square matrices that can have nonzero entries only on and above the main diagonal, whereas any entry below the diagonal must be zero.

$$\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 2 & 2 & 0 \\ 0 & -3 & 5 & 1 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Similarly, lower triangular matrices can have nonzero entries only on and below the main diagonal, whereas any entry above the diagonal must be zero.

Any entry on the main diagonal of a triangular matrix may be zero or not

$$\begin{bmatrix} 5 & 0 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ 2 & 1 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 0 & 0 & 0 \\ 2 & -3 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 1 & 9 & 3 & 5 \end{bmatrix}$$

Diagonal Matrices

These are square matrices that can have nonzero entries only on the main diagonal. Any entry above or below the main diagonal must be zero.

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 6 \end{bmatrix}, \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Scalar Matrix

If all the diagonal entries of a diagonal matrix S are equal, say c, we call S a scalar matrix because multiplication of any square matrix A of the same size by S has same effect as the multiplication by scalar, that is,

$$AS = SA = cA$$

Example:

$$S=\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \text{ and } A=\begin{bmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 2 & 1 & 6 \end{bmatrix} \Rightarrow AS=\begin{bmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 2 & 1 & 6 \end{bmatrix}\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}=\begin{bmatrix} 2c & 3c & 2c \\ 5c & 1c & 4c \\ 2c & 1c & 6c \end{bmatrix}=cA$$

and

$$\Rightarrow SA=\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} 2 & 3 & 2 \\ 5 & 1 & 4 \\ 2 & 1 & 6 \end{bmatrix}=\begin{bmatrix} 2c & 3c & 2c \\ 5c & 1c & 4c \\ 2c & 1c & 6c \end{bmatrix}=cA$$

Unit Matrix (Identity Matrix)

A scalar matrix whose entries on the main diagonal are all 1 is called a unit matrix (identity matrix) and is denoted by I . Thus,

$$I=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } AI=IA=A$$