

Institute for NET/JRF, GATE, IIT-JAM, M.Sc. Entrance, JEST, TIFR and GRE in Physics

(f) Symmetric, Skew-Symmetric, and Orthogonal Matrices

Symmetric Matrix

A real square matrix $A = [a_{ij}]$ is called symmetric if transposition leaves it unchanged,

$$A^T = A$$
, thus $a_{ij} = a_{ji}$.

The Eigen-values of symmetric matrix are always real.

Example:
$$A = \begin{bmatrix} -3 & 1 & 5 \\ 1 & 0 & -2 \\ 5 & -2 & 4 \end{bmatrix}$$

Skew-Symmetric Matrix

A real square matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is called skew-symmetric if transposition gives the negative of A,

$$A^T = -A$$
, thus $a_{ij} = -a_{ji}$.

Every skew-symmetric matrix has all main diagonal entries zero.

The Eigen-values of skew-symmetric matrix are pure imaginary or zero.

Example:
$$A = \begin{bmatrix} 0 & 9 & -12 \\ -9 & 0 & 20 \\ 12 & -20 & 0 \end{bmatrix}$$

NOTE:

Any real square matrix A may be written as the sum of a symmetric matrix R and a skew-symmetric matrix S, where

$$R = \frac{1}{2}(A + A^T)$$
 and $S = \frac{1}{2}(A - A^T) \Rightarrow A = R + S$

Example:
$$A = \begin{bmatrix} 3 & -4 & -1 \\ 6 & 0 & -1 \\ -3 & 13 & -4 \end{bmatrix} = R + S = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 0 & 6 \\ -2 & 6 & -4 \end{bmatrix} + \begin{bmatrix} 0 & -5 & 1 \\ 5 & 0 & -7 \\ -1 & 7 & 0 \end{bmatrix}$$

Orthogonal Matrix

A real square matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is called orthogonal if transposition gives the inverse of A,

$$A^T = A^{-1}$$

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NOTE:

(i) A real square matrix is orthogonal if and only if its column vectors and also its row vectors form an **orthonormal system**, that is

$$a_i.a_j = a_i^T a_j = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

Thus,

$$A^T A = I$$

- (ii) The determinate of an orthogonal matrix has the value +1 or -1.
- (iii) The eigenvalues of an orthogonal matrix A are real or complex conjugate in pairs and have absolute value 1.

Example:
$$A = \begin{bmatrix} 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \\ 1/3 & 2/3 & -2/3 \end{bmatrix}$$

Its characteristic equation is $-\lambda^3 + \frac{2}{3}\lambda^2 + \frac{2}{3}\lambda - 1 = 0 \Rightarrow (\lambda + 1)(\lambda^2 - \frac{5}{3}\lambda + 1) = 0$

$$\Rightarrow \lambda = -1, \quad (5 + i\sqrt{11})/6, \quad (5 - i\sqrt{11})/6$$

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