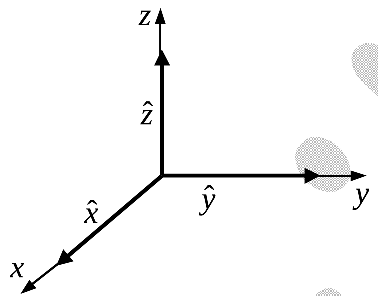
2.Coordinate System

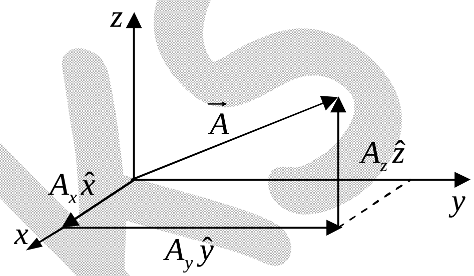
If we want to represent any vector $\vec{A}$ then we need a coordinate system. We have three different coordinate system namely cartesian coordinate system, spherical polar coordinate system and cylindrical polar coordinate system.

2(a). Cartesian Coordinate System

Let $\hat{x}$, $\hat{y}$ and $\hat{z}$ be unit vectors parallel to the x, y and z axis, respectively. An arbitrary vector $\vec{A}$ can be expanded in terms of these basis vectors

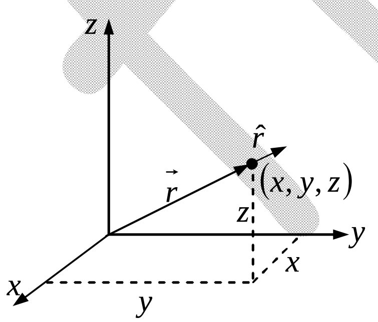
$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

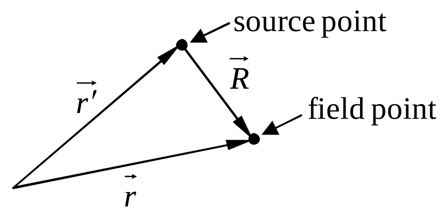




The numbers A_x , A_y , and A_z are called component of $\vec{A}$; geometrically, they are the projections of $\vec{A}$ along the three coordinate axes.

Position and Separation Vectors





The location of a point in three dimensions can be described by listing its Cartesian coordinates (x, y, z). The vector to that point from the origin is called the position vector:

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}.$$

Its magnitude, $r=\sqrt{x^2+y^2+z^2}$ is the distance from the origin,

and $\hat{r}=\frac{\vec{r}}{r}=\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}$ is a unit vector pointing radially outward.

Note: In electrodynamics one frequently encounters problems involving two points- typically, a **source point ,** $\vec{r}'$, where an electric charge is located, and a **field point,** $\vec{r}$ **,** at which we are calculating the electric or magnetic field. We can define **separation vector** from the source point to the field point by $\vec{R}$;

$$\vec{R}=\vec{r}-\vec{r}'.$$

Its magnitude is $R=\left|\vec{r}-\vec{r}'\right|$,

and a unit vector in the direction from $\vec{r}'$ to $\vec{r}$ is $\hat{R}=\frac{\vec{R}}{R}=\frac{\vec{r}-\vec{r}'}{\left|\vec{r}-\vec{r}'\right|}$.

In Cartesian coordinates, $\vec{R}=(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}$

$$\left|\vec{R}\right|=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$\hat{R}=\frac{(x-x')\hat{x}+(y-y')\hat{y}+(z-z')\hat{z}}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

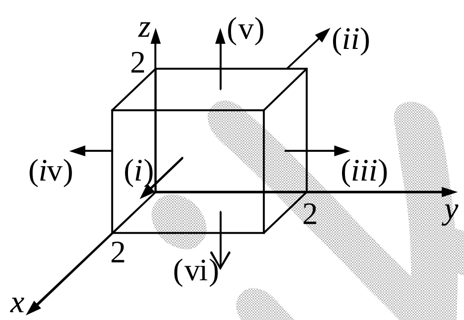
Infinitesimal Displacement Vector $\left(d\vec{l}\right)$

The infinitesimal displacement vector**,** from (x, y, z) to $(x+dx, y+dy, z+dz)$, is

$$d\vec{l} = dx\hat{x} + dy\hat{y} + dz\hat{z}$$

Area Element $\left(d\vec{a}\right)$

For closed surface area element is perpendicular to the surface pointing outwards as shown in figure below.

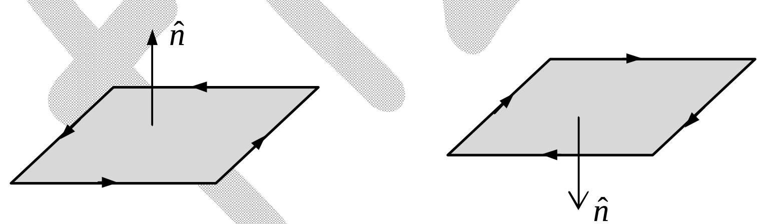


(i) For $x=2$ plane, $d\vec{a} = dydz\hat{x}$ (ii) For $x=0$ plane, $d\vec{a} = -dydz\hat{x}$

(iii) For $y=2$ plane, $d\vec{a} = dxdz\hat{y}$ (iv) For $y=0$ plane, $d\vec{a} = -dxdz\hat{y}$

(v) For $z=2$ plane, $d\vec{a} = dxdy\hat{z}$ (vi) For $z=0$ plane, $d\vec{a} = -dxdy\hat{z}$

For open surface area element is shown in figure below (use right hand rule)



Volume Element $(d\tau)$

Volume element $d\tau = dxdydz$