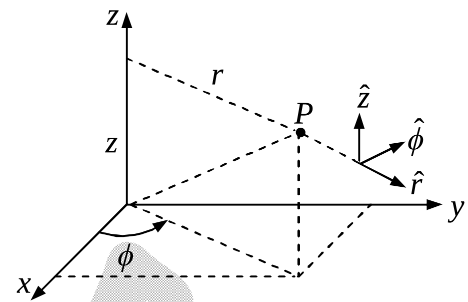
2(c). Cylindrical Polar Coordinate System

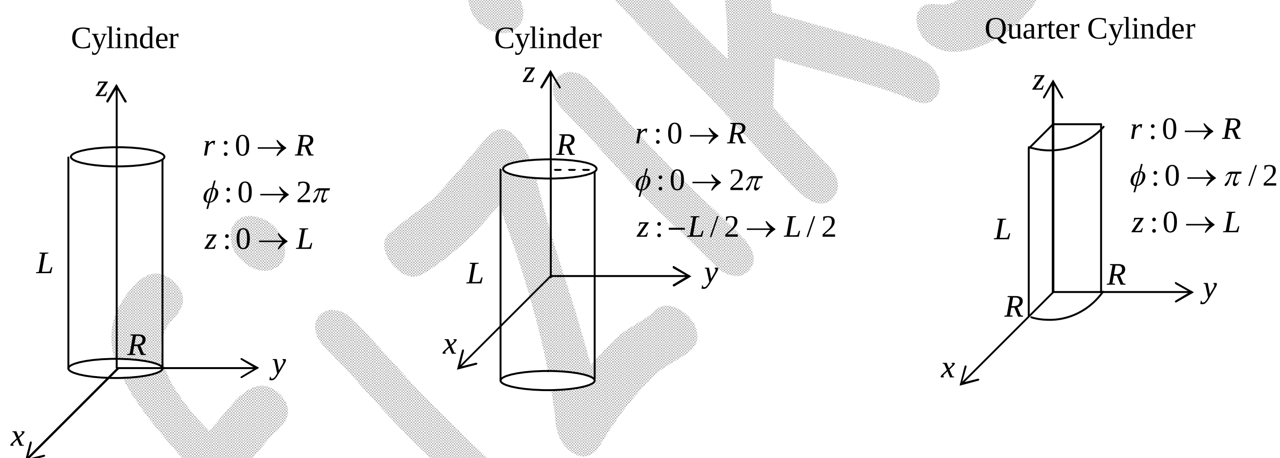
The cylindrical coordinates r, ϕ, z of a point P are defined in figure. Notice that ϕ has the same meaning as in spherical coordinates, and z is the same as Cartesian; r is the distance to P from the z axis, whereas the spherical coordinate r is the distance from the origin. The relation to Cartesian coordinates is



$$x = r\cos\phi, \quad y = r\sin\phi \;, \quad z = z$$

The range of r is $0 \to \infty$, ϕ goes from $0 \to 2\pi$, and z from $-\infty$ to ∞

If we have cylinder or any part of the cylinder, then we can specify r, ϕ and z. Lets consider some examples shown in figure below:



Infinitesimal Displacement Vector $\left(d\vec{l}\right)$

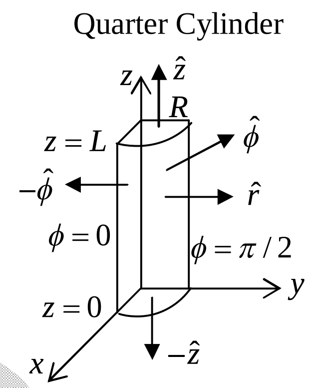
If (r, ϕ, z) changes to $(r + dr, \phi + d\phi, z + dz)$,

The infinitesimal displacements are

$$dl_r = dr, \quad dl_\phi = r d\phi, \quad dl_z = dz,$$

So $$d\vec{l} = dr\,\hat{r} + r\,d\phi\hat{\phi} + dz\,\hat{z}$$

Area Element $\left(d\vec{a}\right)$



If we are integrating over the surface of an quarter cylinder, for instance, then r is constant $(r = R)$, whereas ϕ and z change, so,

$$d\vec{a}_1 = dl_\phi dl_z \hat{r} = r d\phi dz \hat{r}.$$

If the surface lies in the xy plane, then z is constant ($z = L$) while r and ϕ vary, then

$$d\vec{a}_2 = dl_r dl_\phi \hat{z} = r dr d\phi \hat{z}$$

If the surface lies in the xy plane, then z is constant ($z = 0$) while r and ϕ vary, then

$$d\vec{a}_2 = -dl_r dl_\phi \hat{\theta} = -r dr d\phi \hat{z}$$

If the surface lies in the yz plane, then ϕ is constant ($\phi = \pi/2$) while r and z vary, then

$$d\vec{a}_3 = dl_r dl_z \hat{\phi} = dr dz \hat{\phi}$$

If the surface lies in the xz plane, then ϕ is constant ($\phi = 0$) while r and z vary, then

$$d\vec{a}_4 = -dl_r dl_z \hat{\phi} = -dr dz \hat{\phi}$$

Volume Element $(d\tau)$

The infinitesimal volume element $d\tau$, in cylindrical coordinates, is the product of the three infinitesimal displacements:

$$d\tau = dl_r dl_\phi dl_z = r dr d\phi dz$$