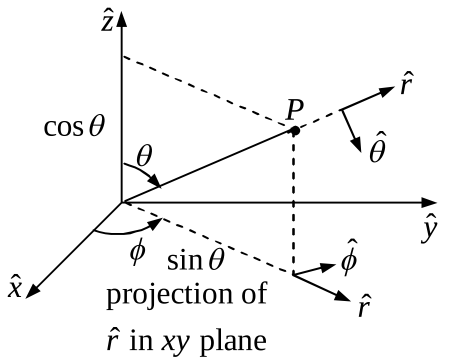
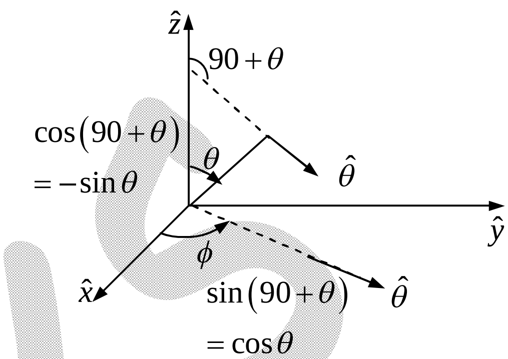
2(d).Transformation of a Vector from Cartesian to Spherical Polar

We can transform any vector $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ in Cartesian coordinates to Spherical polar coordinate as $\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$.





Thus

$$A_r = \vec{A}.\hat{r} = A_x(\hat{x}.\hat{r}) + A_y(\hat{y}.\hat{r}) + A_z(\hat{z}.\hat{r})$$

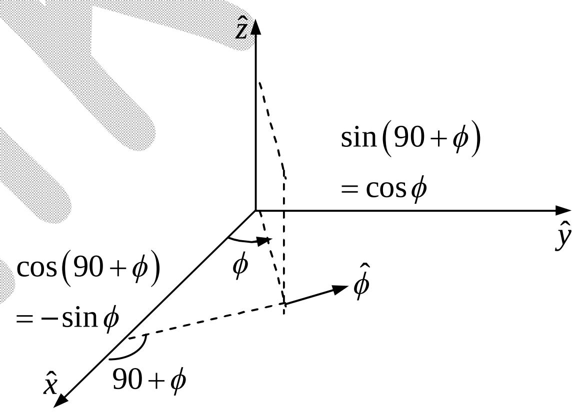
$$A_\theta = \vec{A}.\hat{\theta} = A_x(\hat{x}.\hat{\theta}) + A_y(\hat{y}.\hat{\theta}) + A_z(\hat{z}.\hat{\theta})$$

$$A_\phi = \vec{A}.\hat{\phi} = A_x(\hat{x}.\hat{\phi}) + A_y(\hat{y}.\hat{\phi}) + A_z(\hat{z}.\hat{\phi})$$

where

$x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$

and use table given below:



	$\hat{r}$	$\hat{\theta}$	$\hat{\phi}$
$\hat{x}.$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\hat{y}.$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\hat{z}.$	$\cos\theta$	$-\sin\theta$	0

Example: Transform vector $\vec{A} = x\hat{x} + y\hat{y} + z\hat{z}$ in Cartesian coordinates to Spherical polar coordinate as $\vec{A} = A_r\hat{r} + A_\theta\hat{\theta} + A_\phi\hat{\phi}$.

Solution:

$$A_r = \vec{A}.\hat{r} = x(\hat{x}.\hat{r}) + y(\hat{y}.\hat{r}) + z(\hat{z}.\hat{r})$$

$$\Rightarrow A_r = r\sin\theta\cos\phi(\sin\theta\cos\phi) + r\sin\theta\sin\phi(\sin\theta\sin\phi) + r\cos\theta(\cos\theta)$$

$$\Rightarrow A_r = r\sin^2\theta\left(\cos^2\phi + \sin^2\phi\right) + r\cos^2\theta = r$$

$$A_\theta = \vec{A}.\hat{\theta} = x\left(\hat{x}.\hat{\theta}\right) + y\left(\hat{y}.\hat{\theta}\right) + z\left(\hat{z}.\hat{\theta}\right)$$

$$\Rightarrow A_\theta = r\sin\theta\cos\phi(\cos\theta\cos\phi) + r\sin\theta\sin\phi(\cos\theta\sin\phi) + r\cos\theta(-\sin\theta)$$

$$\Rightarrow A_\theta = r\sin\theta\cos\theta\left(\cos^2\phi + \sin^2\phi\right) - r\sin\theta\cos\theta = 0$$

$$A_\phi = \vec{A}.\hat{\phi} = x\left(\hat{x}.\hat{\phi}\right) + y\left(\hat{y}.\hat{\phi}\right) + z\left(\hat{z}.\hat{\phi}\right)$$

$$\Rightarrow A_\phi = r\sin\theta\cos\phi(-\sin\phi) + r\sin\theta\sin\phi(\cos\phi) + r\cos\theta\times 0 = 0$$

Thus in spherical polar coordinate $\vec{A} = r\,\hat{r}$.

Example: Transform vector $\vec{A} = r\,\hat{r}$ in Spherical polar coordinate to Cartesian coordinates to as $\vec{A} = A_x\,\hat{x} + A_y\,\hat{y} + A_z\,\hat{z}$.

Solution:

$$A_x = \vec{A}.\hat{x} = r(\hat{r}.\hat{x}) = r\sin\theta\cos\phi = x$$

$$A_y = \vec{A}.\hat{y} = r(\hat{r}.\hat{y}) = r\sin\theta\sin\phi = y$$

$$A_z = \vec{A}.\hat{z} = r(\hat{r}.\hat{z}) = r\cos\theta = z$$

Thus in cartesian coordinate $\vec{A} = x\hat{x} + y\hat{y} + z\hat{z}$.