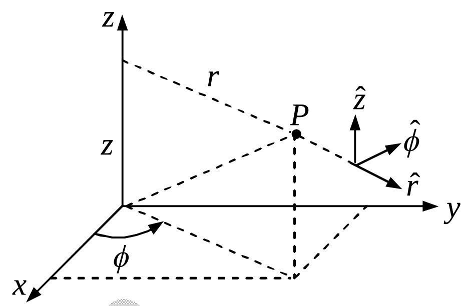
2(e).Transformation of a Vector from Cartesian to Cylindrical Coordinate

We can transform any vector $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ in Cartesian coordinates to cylindrical coordinates as $\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + A_z\hat{z}$



Thus

$$A_r = \vec{A}.\hat{r} = A_x(\hat{x}.\hat{r}) + A_y(\hat{y}.\hat{r}) + A_z(\hat{z}.\hat{r})$$

$$A_\phi = \vec{A}.\hat{\phi} = A_x(\hat{x}.\hat{\phi}) + A_y(\hat{y}.\hat{\phi}) + A_z(\hat{z}.\hat{\phi})$$

$$A_z = \vec{A}.\hat{z} = A_x(\hat{x}.\hat{z}) + A_y(\hat{y}.\hat{z}) + A_z(\hat{z}.\hat{z})$$

where $x = r\cos\phi,\quad y = r\sin\phi,\quad z = z$

use table given below:

	$\hat{r}$	$\hat{\phi}$	$\hat{z}$
$\hat{x}.$	$\cos\phi$	$-\sin\phi$	0
$\hat{y}.$	$\sin\phi$	$\cos\phi$	0
$\hat{z}.$	0	0	1

Example: Transform vector $\vec{A} = x\hat{x} + y\hat{y} + z\hat{z}$ in Cartesian coordinates to cylindrical coordinate as $\vec{A} = A_r\hat{r} + A_\phi\hat{\phi} + A_z\hat{z}$.

Solution:

$$A_r = \vec{A}.\hat{r} = x(\hat{x}.\hat{r}) + y(\hat{y}.\hat{r}) + z(\hat{z}.\hat{r}) \Rightarrow A_r = r\cos\phi(\cos\phi) + r\sin\phi(\sin\phi) + z\times 0 = r$$

$$A_\phi = \vec{A}.\hat{\phi} = x(\hat{x}.\hat{\phi}) + y(\hat{y}.\hat{\phi}) + z(\hat{z}.\hat{\phi}) \Rightarrow A_\phi = r\cos\phi(-\sin\phi) + r\sin\phi(\cos\phi) + z\times 0 = 0$$

$$A_z = \vec{A}.\hat{z} = x(\hat{x}.\hat{z}) + y(\hat{y}.\hat{z}) + z(\hat{z}.\hat{z}) \Rightarrow A_z = r\cos\phi\times 0 + r\sin\phi\times 0 + z\times 1 = z$$

Thus in spherical polar coordinate $\vec{A} = r\hat{r} + z\hat{z}$.

Example: Transform vector $\vec{A} = r\hat{r} + z\hat{z}$ in cylindrical coordinate to Cartesian coordinates to as $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$.

Solution $A_x = \vec{A}.\hat{x} = r(\hat{r}.\hat{x}) + z(\hat{z}.\hat{x}) = r\cos\phi = x$, $A_y = \vec{A}.\hat{y} = r(\hat{r}.\hat{y}) + z(\hat{z}.\hat{y}) = r\sin\phi = y$

$$A_z = \vec{A}.\hat{z} = r(\hat{r}.\hat{z}) + z(\hat{z}.\hat{z}) = z$$

Thus in cartesian coordinate $\vec{A} = x\hat{x} + y\hat{y} + z\hat{z}$.