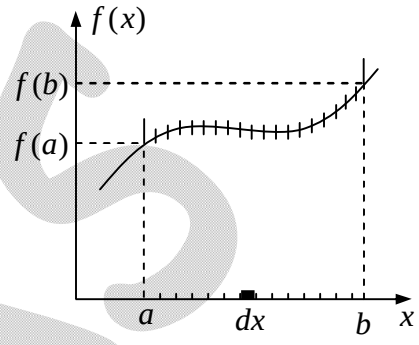
4(d). The Fundamental Theorem of Calculus

Suppose $f(x)$ is a function of one variable. The **fundamental theorem of calculus** states:

$$\int_a^b \frac{df}{dx}\,dx = f(b) - f(a) \quad \text{or} \quad \int_a^b F(x)dx = f(b) - f(a)$$

where $df/dx = F(x)$.

Geometrical Interpretation



According to equation $df = (df/dx)\,dx$ is the infinitesimal change in f when one goes from (x) to $(x + dx)$. The fundamental theorem says that if you chop the interval from a to b into many tiny pieces, dx, and add up the increments df from each little piece, the result is equal to the total change in f is $f(b) - f(a)$.

In other words, there are two ways to determine the total change in the function: either subtract the values at the ends or go step-by-step, adding up all the tiny increments as you go. You'll get the same answer either way.