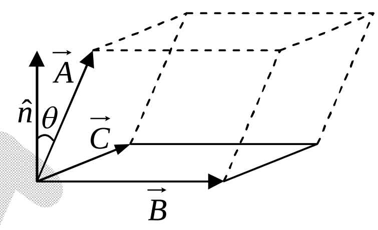
1(c). Triple Products

Since the cross product of two vectors is itself a vector, it can be dotted or crossed with a third vector to form a triple product.

(i) Scalar triple product: $\vec{A}.\left(\vec{B}\times\vec{C}\right)$



Geometrically $\left|\vec{A}.\left(\vec{B}\times\vec{C}\right)\right|$ is the volume of the parallelepiped generated by $\vec{A}$, $\vec{B}$ and $\vec{C}$, since $\left|\vec{B}\times\vec{C}\right|$ is the area of the base, and $\left|\vec{A}\cos\theta\right|$ is the altitude. Evidently,

$$\vec{A}.\left(\vec{B}\times\vec{C}\right)=\vec{B}.\left(\vec{C}\times\vec{A}\right)=\vec{C}.\left(\vec{A}\times\vec{B}\right)$$

In component form $\vec{A}.\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix}A_x & A_y & A_z\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix}$

Note that the dot and cross can be interchanged: $\vec{A}.\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\times\vec{B}\right).\vec{C}$

(ii) Vector triple product: $\vec{A}\times\left(\vec{B}\times\vec{C}\right)$

The vector triple product can be simplified by the so-called **BAC-CAB** rule:

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\vec{B}\left(\vec{A}.\vec{C}\right)-\vec{C}\left(\vec{A}.\vec{B}\right)$$