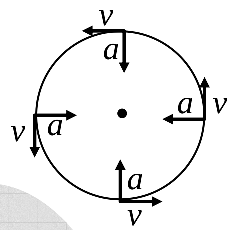
(c) Circular Motion

For circular motion for radius r_0, at $r = r_0$, then $\ddot{r} = 0$ so $F_r = ma_r = -mr_0\dot{\theta}^2$, where F_r is force in radial direction and $F_\theta = ma_\theta = mr_0\ddot{\theta}$, where F_θ is force in tangential direction .

There are two type of circular motion

1.1 Uniform Circular Motion

If there is not any force in tangential direction $F_\theta = 0$ is condition, then motion is uniform circular motion i.e., $\dot{\theta} = \omega$ is constant known as angular speed and tangential speed is given by $v = r_0\omega$.

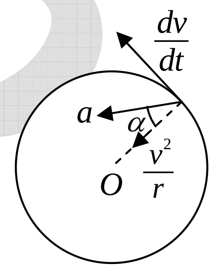


1.2 Non-uniform Circular Motion

For nonuniform circular motion of radius r radial acceleration is

$a_r = -mr\dot{\theta}(t)^2 = -mr\omega(t)^2 = -\dfrac{mv(t)^2}{r}$ and

tangential acceleration $a_\theta = \dfrac{dv}{dt}$



the acceleration for non uniform circular motion is given by $\vec{a} = a_r\hat{r} + a_\theta\hat{\theta} \Rightarrow -\dfrac{mv^2}{r}\hat{r} + \dfrac{dv}{dt}\hat{\theta}$

the magnitude of acceleration is given by $|a| = \sqrt{a_r^2 + a_\theta^2} \Rightarrow \sqrt{\left(\dfrac{mv^2}{r}\right)^2 + \left(\dfrac{dv}{dt}\right)^2}$

Example: Find the magnitude of the linear acceleration of a particle moving in a circle of radius $10\,cm$ with uniform speed completing the circle in $4\,s$.

Solution: The distance covered in completing the circle is $2\pi r = 2\pi \times 10\,cm$. The linear speed is

$$v = \frac{2\pi r}{t} = \frac{2\pi \times 10\,cm}{4\,s} = 5\pi\,cm/s$$

The linear acceleration is

$$a = \frac{v^2}{r} = \frac{(5\pi\,cm/s)^2}{10\,cm} = 2.5\pi^2 cm/s^2$$

This acceleration is directed towards the centre of the circle.

Calculation of torque and angular momentum

$$\vec{F}=\vec{F}_r\hat{r}+\vec{F}_\theta\hat{\theta}$$

Torque is given by $\vec{\tau}=\vec{r}\times\vec{F}=r\hat{r}\times\vec{F}=m\left(r^2\ddot{\theta}+2r\dot{r}\dot{\theta}\right)\hat{r}\times\hat{\theta}=m\left(r^2\ddot{\theta}+2r\dot{r}\dot{\theta}\right)\hat{z}$

Torque is also defined rate of change of momentum $\tau=\frac{dJ}{dt}=m\left(r^2\ddot{\theta}+2r\dot{r}\dot{\theta}\right)=\frac{d\left(mr^2\dot{\theta}\right)}{dt}$

So, angular momentum is given by $J=mr^2\dot{\theta}$

If $F_\theta=0\Rightarrow m\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)=0\Rightarrow\frac{d\left(mr^2\dot{\theta}\right)}{dt}=0\Rightarrow mr^2\dot{\theta}=\text{constant}$

If there is not any tangential force is in plane then angular momentum of the system is conserve.