

Newton's Laws of Motion

Three physical laws laid the foundation of classical mechanics. They describe the relationship between a body and the forces acting upon it and its motion in response to those forces.

1. First Law (Law of Inertia)

Inertia is an inherent property of matter that always opposes the change of its state. In general mass is measure of inertia. Heavier the mass greater is the inertia. Inertial frame is a coordinate system with a clock attached that moves with a constant speed.

In the absence of an external force in the inertial frame an object either remains at rest or continues to move at a constant velocity.

According to first law if net force (the vector sum of all forces acting on an object) is zero, then the momentum of the object is constant (Both magnitude and direction). So,

the first law can be mathematically stated as $\sum \vec{F} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0$.

2. Second Law

The second law states that the net force on an object is equal to the rate of change (that is, the *derivative*) of its linear momentum \vec{p} in an inertial reference frame i.e.

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$$

If the mass is constant then the vector sum of the external forces \vec{F} on an object is equal to the mass m of that object multiplied by the acceleration vector \vec{a} of the object $\vec{F} = m\vec{a}$.

$$\vec{F} = m \frac{d\vec{v}}{dt} = m\vec{a}$$

One can visualize Newton's second law as cause and effect phenomenon where external force is equivalent to cause and resulting acceleration is its effect which is measured by a force.

In the case when the velocity is very high (close to velocity of light) Newton's law should be modified according to special theory of relativity.

3. Third Law

To every action there is an equal and opposite reaction. The action and reaction acts on two different bodies. Consider two bodies body one exerts a force (\vec{F}_{12}) on second body and the second body simultaneously exerts a force (\vec{F}_{21}) equal in magnitude and opposite in direction on the first body i.e. $\vec{F}_{12} = -\vec{F}_{21}$

4. Impulse

An impulse \vec{I} occurs when a force \vec{F} acts over a short interval of time Δt ($\Delta t \rightarrow 0$) and it is given by $\vec{I} = \int \vec{F} dt$. Since the force is the time derivative of momentum it follows that

$$\vec{I} = \Delta\vec{p} \text{ when mass is constant } \vec{I} = m\Delta\vec{v}$$

5. Equation of Motion

The equation of motion is given by $\sum F_x = ma_x, \sum F_y = ma_y$. Number of equations must be equal to the number of unknown variables so that one can solve them for complete as well unique solution.