1(c). Spin and Magnetic Moment

Proton and neutrons, like electrons, are fermions with spin quantum numbers of $s=\frac{1}{2}$.

This means they have spin angular momenta $\vec{S}$ of magnitude

$$\left|\vec{S}\right|=\sqrt{s(s+1)}\hbar=\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar=\frac{\sqrt{3}}{2}\hbar$$

and spin magnetic quantum number of $m_s=\pm\frac{1}{2}$.

As in the case of electrons, magnetic moments are associated with the spins of protons and neutrons. In nuclear physics magnetic moments are expressed in **nuclear magnetons** (μ_N), where

Nuclear magneton $\mu_N=\frac{e\hbar}{2m_p}=5.051\times10^{-27}\,J/T=3.152\times10^{-8}\,eV/T$ where m_p is the proton mass.

In atomic physics we have defined **Bohr magneton** $\mu_B=\frac{e\hbar}{2m_e}$ where m_e is the electron mass.

The nuclear magneton is smaller than the Bohr magneton by the ratio of the proton mass to the electron mass which is 1836.

$$\frac{\mu_B}{\mu_N}=\frac{m_p}{m_e}=1836.$$

The spin magnetic moments of the proton and neutron have components in any direction of

Proton $\mu_{pz}=\pm2.793\mu_N$

Neutron $\mu_{nz}=\mp1.913\mu_N$

There are two possibilities for the signs of μ_{pz} and μ_{nz}, depending on whether m_s is $-\frac{1}{2}$ or $+\frac{1}{2}$. The $\pm$ sign is used for μ_{pz} because $\vec{\mu}_{pz}$ is in the same direction as the spin $\vec{S}$, whereas $\mp$ is used for μ_{nz} because $\vec{\mu}_{nz}$ is opposite to spin $\vec{S}$.

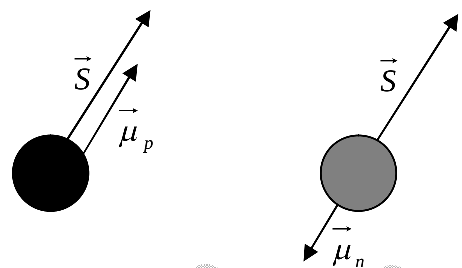

Figure: Spin magnetic moment ($\vec{\mu}$) and spin angular momentum ($\vec{S}$) directions for neutron and protons.

Note: For neutron, magnetic moment is expected to be zero as $e = 0$ but $\vec{\mu}_{nz} = \mp 1.913\mu_N$. At first glance it seems odd that the neutron, with no net charge, has spin magnetic moment. But if we assume that the neutron contains equal amounts of positive and negative charge, a spin magnetic moment arise if these charges are not uniformly distributed. Thus we can say that neutron has physical significance of negative charges because magnetic moment is opposite to that of its intrinsic spin angular momentum.