1(f). Binding Energy

When nuclear masses are measured, it is found that they are less than the sum of the masses of the neutrons and protons of which they are composed. This is in agreement with Einstein's theory of relativity, according to which the mass of a system bound by energy B is less than the mass of its constituents by B/c^2 (where c is the velocity of light).

The Binding energy B of a nucleus is defined as the difference between the energy of the constituent particles and of the whole nucleus. For a nucleus of atom ${}_Z^A X$,

$$B = \left[ZM_p + NM_N - {}_Z^A M \right] c^2 = [ZM_H + NM_N - M({}_Z^A X)]c^2$$

If mass is expressed in atomic mass unit

$$B = \left[ZM_p + NM_N - {}_Z^A M \right] \times 931.5\, MeV = [ZM_H + NM_N - M({}_Z^A X)] \times 931.5\, MeV$$

M_p : Mass of free proton, M_N : M_N: Mass of free neutron,

M_H : mass of hydrogen atom ${}_Z^A M$: mass of the nucleus,

Z : Number of proton, N : Number of neutron,

$M\left({}_Z^A X\right)$: mass of atom.

9.1.5.1 Binding Energy per Nucleon

The **binding energy per nucleon** for a given nucleus is found by dividing its total binding energy by the number of nucleon it contains. Thus binding energy per nucleon is

$$\frac{B}{A} = \frac{c^2}{A}\left[ZM_p + NM_N - {}_Z^A M \right] = \frac{c^2}{A}[ZM_H + NM_N - M({}_Z^A X)]$$

The binding energy per nucleon for ${}_1^2\text{H}$ is $\frac{2.224}{2} = 1.112 \text{ MeV / nucleon}$ and for ${}_{63}^{209}\text{Bi}$ it is $\frac{1640 \text{ MeV}}{209} = 7.8 \text{ MeV / nucleon}$.

Figure below shows the binding energy per nucleon against the number of nucleons in various atomic nuclei.

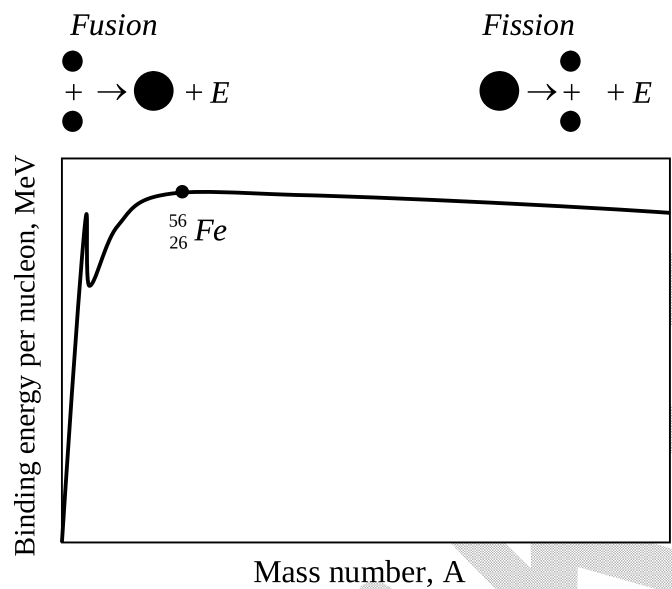

Figure: Binding energy per nucleon as function of mass number.

The greater the binding energy per nucleon, the more stable the nucleus is. The graph has the maximum of 8.8 MeV / nucleon when the number of nucleons is 56. The nucleus that has 56 protons and neutrons is ${}^{56}_{26}\text{Fe}$ an iron isotope. This is the most stable nucleus of them all, since the most energy is needed to pull a nucleon away from it.

Two remarkable conclusions can be drawn from the above graph.

(i) If we can somehow split a heavy nucleus into two medium sized ones, each of the new nuclei will have more binding energy per nucleon than the original nucleus did. The extra energy will be given off, and it can be a lot. For instance, if the uranium nucleus ${}^{235}_{92}\text{U}$ is broken into two smaller nuclei, the binding energy difference per nucleon is about 0.8 MeV. The total energy given off is therefore

$$\left(0.8\,\frac{\text{MeV}}{\text{nucleon}}\right)(235\ \text{nucleon}) = 188\,\text{MeV}$$

This process is called as **nuclear fission**.

(ii) If we can somehow join two light nuclei together to give a single nucleus of medium size also means more binding energy per nucleon in the new nucleus. For instance, if two ${}_1^2\text{H}$ deuterium nuclei combine to form a ${}_2^4\text{He}$ helium nucleus, over 23 MeV is released. Such a process, called **nuclear fusion**, is also very effective way to obtain energy. In fact, nuclear fusion is the main energy source of the sun and other stars.

Example: The measured mass of deuteron atom $\left({}_1^2\text{H}\right)$, Hydrogen atom $\left({}_1^1\text{H}\right)$, proton and neutron is $2.01649\ u$, $1.00782\ u$, $1.00727\ u$ and $1.00866\ u$. Find the binding energy of the deuteron nucleus (unit $MeV/nucleon$).

Solution: Here $A = 2,\ Z = 1,\ N = 1$

$$B.E. = [ZM_H + NM_N - M({}_1^2H)] \times 931.5\ MeV$$

$$= [1\times 1.00782 + 1\times 1.00866 - 2.01649]\times 931.5\ MeV$$

$$= [0.00238]\times 931.5\ MeV = 2.224\ MeV$$

Example: The binding energy of the neon isotope ${}_{10}^{20}\text{Ne}$ is 160.647 MeV. Find its atomic mass.

Solution: Here $A = 10,\ Z = 10,\ N = 10$

$$\text{M}\ ({}_{\text{Z}}^{\text{A}}\text{X}) = \left[\text{ZM}_\text{H} + \text{NM}_\text{N}\right] - \frac{\text{B}}{931.5\ \text{MeV/u}}$$

$$\text{M}\ ({}_{10}^{20}\text{Ne}) = \left[10(1.00782) + 10(1.00866)\right] - \frac{160.647}{931.5\ \text{MeV/u}} = 19.992\ \text{u}$$

Example:

(a) Find the energy needed to remove a neutron from the nucleus of the calcium isotope ${}_{20}^{42}Ca$.

(b) Find the energy needed to remove a proton from this nucleus.

(c) Why are these energies different?

Given: atomic masses of ${}_{20}^{42}Ca = 41.958622u$, ${}_{20}^{41}Ca = 40.962278u$, ${}_{19}^{41}K = 40.961825u$, and mass of ${}_0^1 n = 1.008665u$, ${}_1^1 p = 1.007276u$.

Solution:

(a) $^{42}_{20}Ca \rightarrow ^{41}_{20}Ca + ^{1}_{0}n$ **;**

Total mass of the $^{41}_{20}Ca$ *and* $^{1}_{0}n = 41.970943\ u$

Mass defect $\Delta m = 41.970943 - 41.958622 = 0.012321\,u$

So, B.E. of missing neutron $= \Delta m \times 931.5 = 11.48\ MeV$

(b) $^{42}_{20}Ca \rightarrow ^{41}_{19}K + ^{1}_{1}p$ **;**

Total mass of the $^{41}_{19}K$ *and* $^{1}_{1}p = 41.919101\ u$

Mass defect $\Delta m = 41.919101 - 41.958622 = 0.010479\,u$

So, B.E. of missing protron $= \Delta m \times 931.5 = 10.27\ MeV$

(c) The neutron was acted upon only by attractive nuclear forces whereas the proton was also acted upon by repulsive electric forces that decrease its binding energy.