(a) Liquid Drop Model of Nucleus (Semi-Empirical Mass Formula or Weizsacker Formula)

Using semi-empirical approach (based on experimental results) Weizsacker showed that it is possible to achieve a quantitative and more basic understanding of binding energies of nuclei.

Assumptions

(i) Nucleus is modeled on a drop of liquid.

(ii) The nuclear interaction between protons and neutrons, between protons and protons, and between neutron and neutrons are identical

(iii) $N = Z = \frac{A}{2}$

(iv) Nuclear forces are saturated.

Deduction

(i) Volume Energy term (B_V)

In a liquid drop, in which each molecule interacts only with its neighbors and number of neighboring molecules is independent of overall size of the liquid drop, the binding energy of liquid drop is $B = LM_m A$ where $L =$ latent heat of liquid, $M_m =$ mass of each molecule, $A =$ number of molecules.

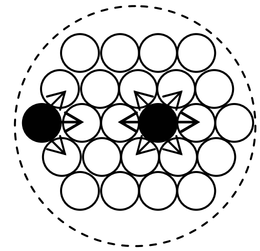
In analogy to the liquid drop, for nuclei we expect a volume term in the expression for binding energy.

Volume energy term, $B_V = a_V A$, where a_V is volume coefficient ($= 14.1 MeV$) and A is mass number.

(ii) Surface Energy Term (B_s)

At the surface of the nucleus, there are nucleons which are not surrounded from all sides; consequently these surface nucleons are not bound as tightly as the nucleons in the interior and hence its binding energy is less. The larger the nucleus, the smaller the proportion of nucleons at the surface.

Surface area of the nucleus = $4\pi R^2 = 4\pi R_0^2 A^{2/3}$.



Hence number of nucleons with fewer than maximum number of neighbor is proportional to $A^{2/3}$. So reducing the binding energy by introducing the term

$$B_s = -a_s A^{2/3}$$

where a_s is surface energy coefficient ($=13.0\,MeV$).

It is most significant for lighter nuclei since a greater fraction of their nucleons are on the surface. Because natural systems always tend to evolve toward configurations of minimum potential energy, nuclei tend toward configurations of maximum binding energy. Hence a nucleus should exhibit the same surface-tension effects as a liquid drop, and in the absence of other effects it should be spherical, since a sphere has the least surface area for a given volume.

(iii) Coulomb Energy term (B_c)

The electric repulsion between each pair of proton in a nucleus also contributes towards decreasing its binding energy. The potential energy of protons 'r' apart is equal to

$$V = \frac{e^2}{4\pi\varepsilon_o r}.$$

Since there are $\frac{Z(Z-1)}{2}$ pair of protons, the coulomb energy $B_c = \frac{Z(Z-1)}{2}V$,

$$B_c = \frac{e^2}{4\pi\varepsilon_o}\frac{Z(Z-1)}{2}\left(\frac{1}{r}\right)_{av.}$$

Now, $\left(\frac{1}{r}\right)_{av.}$ is average value of $\left(\frac{1}{r}\right)$, averaged over all proton pairs. If the protons are uniformly distributed $\left(\frac{1}{r}\right)_{av.} \propto \frac{1}{R} \propto \frac{1}{A^{2/3}}$, thus $B_c = -a_c \frac{Z(Z-1)}{A^{1/3}}$

Where a_c is Coulomb energy coefficient ($=0.595\,MeV$)

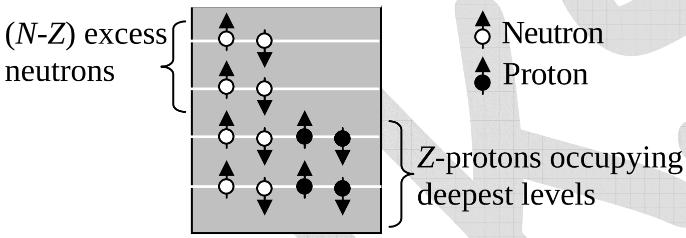
The coulomb energy is negative because it arises from an effect that opposes nuclear stability. So, the total Binding energy is $B = B_v + B_s + B_c = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}}$.

(iv) Corrections to the Formula

The above binding energy formula can be improved by taking into account two effects that do not fit into the simple liquid drop model but which make sense in terms of a model that provides for nuclear energy levels. The above result was improved by including two effects

(a)Asymmetry Effect (B_a)

Asymmetry Energy Term, B_a depends on the neutron excess $(N-Z)$ and decreases the nuclear binding energy. So far, we have neglected the quantization of energy states of individual nucleons in the nucleus and the application of the Pauli Exclusion Principle.



If we put Z protons and N neutrons into the nuclear energy shells, the lowest Z energy levels are filled first. By Pauli Exclusion Principle, the excess $(N-Z)$ neutrons must go into previously unoccupied quantum states since the first Z quantum states are already filled up with protons and neutrons.

These $(N-Z)$ excess neutrons are occupying higher energy quantum states and are consequently less tightly bound than the first $2Z$ nucleons which occupy the deepest lying energy levels. Thus neutron asymmetry gives rise to a disruptive term in nuclear binding energy. Excess energy per nucleon $\alpha \frac{N-Z}{A}$.

Since the total number of excess neutrons is $(N-Z)$, the total deficit in nuclear binding energy is proportional to product of these

$$\Rightarrow B_a = -a_a \frac{(N-Z)^2}{A} = -a_a \frac{(A-2Z)^2}{A},$$

where a_a is asymmetric energy coefficient (19.0 MeV).

(b) Pairing Effect

Since all the previous terms have involved a smooth variation of B whenever Z or N changes and does not account for the kinks which show an evidence for favored pairing.

In liquid drop model we have omitted the intrinsic spin of the nucleons and shell effects. This is corrected by adding a pairing energy term B_P to the nuclear binding energy.

$$B_p = (\pm, 0)\frac{a_p}{A^{+3/4}}, \; a_p = \begin{cases} 0 \text{ for odd-even or even-odd} \\ \text{negative for odd-odd} \\ \text{positive for even-even} \end{cases} \quad \text{and } a_p = 33.5\,MeV$$

The final expression for binding energy is

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A}(\pm, 0)\frac{a_p}{A^{3/4}}$$

Now, nuclear mass can be written as

$$M(Z, A) = AM_N - Z(M_N - M_P) - B/c^2 \qquad (M \,\&\, B \text{ in mass units})$$

$$M(Z, A) = AM_N - Z(M_N - M_P) + \left\{-a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{\left(A-2Z\right)^2)}{A}(\mp, 0)\, a_p A^{-3/4}\right\}\frac{1}{c^2}$$