**(c) Mass Parabolas**

From the semi-empirical mass equation we have

$$M(Z,A) = AM_N - Z(M_N - M_P) - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_a \frac{(A-2Z)^2}{A} (\mp, 0) a_p A^{-3/4}$$

$$M(Z,A) = A\left[M_N - \left(a_v - a_a - \frac{a_s}{A^{1/3}}\right)\right] + Z\left[(M_P - M_N) - \frac{a_c}{A^{1/3}} - 4a_a\right] + Z^2\left[\frac{a_c}{A^{1/3}} + \frac{4a_a}{A}\right] \pm E_p$$

or $M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta$

where $\alpha = M_N - \left(a_v - a_a - \frac{a_s}{A^{1/3}}\right)$, $\beta = -4a_a - (M_n - M_p) - \frac{a_c}{A^{1/3}}$ and $\gamma = \left(\frac{4a_a}{A} + \frac{a_c}{A^{1/3}}\right)$.

$\delta$ ıs pairing energy $(E_P) = +\delta$ for even $Z$ even $N$

$= 0$ for odd $Z$ even $N$ or even $N$ and odd $Z$

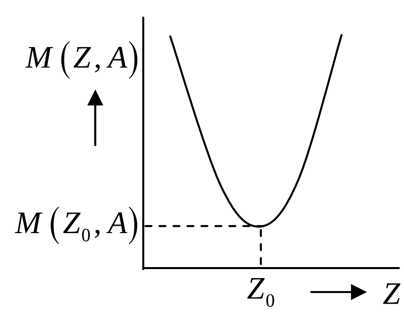
$= -\delta$ for odd $Z$ odd $N$

When $A$ is constant, the equation $M(Z,A) = \alpha A + \beta Z + \gamma Z^2 \pm \delta$ represents a parabola. Thus the plot of $M$ and $Z$ is parabolic with the "minimum" corresponding to that value of $Z$ which gives the (hypothetical) "most stable" isobar in the isobaric family.

**For Odd** $A\ (\delta = 0)$

As either one of $N$ or $Z$ is even and the other one is odd (since odd + even = odd), so only one parabola implying that there is only one stable nucleus.

Consider the isobaric family for $A$,




$$\left(\frac{\delta M}{\delta Z}\right)_A = \beta + 2\gamma Z_0 = 0,$$

{ $Z_0 =$ Nuclear charge of "most stable nuclei},

$$\therefore Z_0 = \frac{-\beta}{2\gamma} \Rightarrow (\beta = -2\gamma Z_0).$$

So mass of the "most stable" isobar is

$$M(Z_0,A) = \alpha A - 2\gamma Z_0 Z_0 + \gamma Z_0^2 \quad (\because \beta = -2\gamma Z_0)$$

$$\therefore M(Z_0,A) = \alpha A - \gamma Z_0^2$$

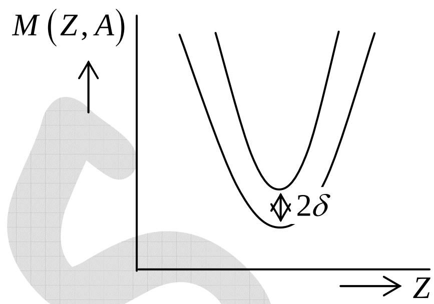
Also, $M(Z,A) = \alpha A - 2\gamma Z_0 . Z + \gamma Z^2$

The difference in masses for odd $A$ is:

$$M(Z,A) - M(Z_0,A) = -2\gamma Z_0 Z + \gamma Z^2 + \gamma Z_0^{\,2} = \gamma (Z - Z_0)^2 = \gamma (Z - Z_0)^2$$

**Even $A$ isobars ($\delta \neq 0$)**

Here pairing term $\delta \neq 0$ since both odd-odd and even-even nuclei are included. So two parabolas,




For odd-odd: $M(Z_0,A) = \alpha A - \gamma Z_0^{\,2} - \delta$

For even-even: $M(Z_0,A) = \alpha A - \gamma Z_0^{\,2} + \delta$

where $Z_0 = -\dfrac{\beta}{2\gamma}$

The vertical separation between two parabolas is $2\delta$

$$M(Z,A) = \alpha A - 2\gamma Z_0 Z + \gamma Z^2 \pm \delta$$