(e) Shell Model of Nucleus

Salient Features of Single Particle Shell Model

(i) Each nucleon experiences a central attractive force which can be as ascribed to the average effect of the remaining (A-1) nucleons in the nucleus.

(ii) In this central field, each nucleon moves in a shell depending on its energy and angular momentum in a manner analogous to atomic orbitals.

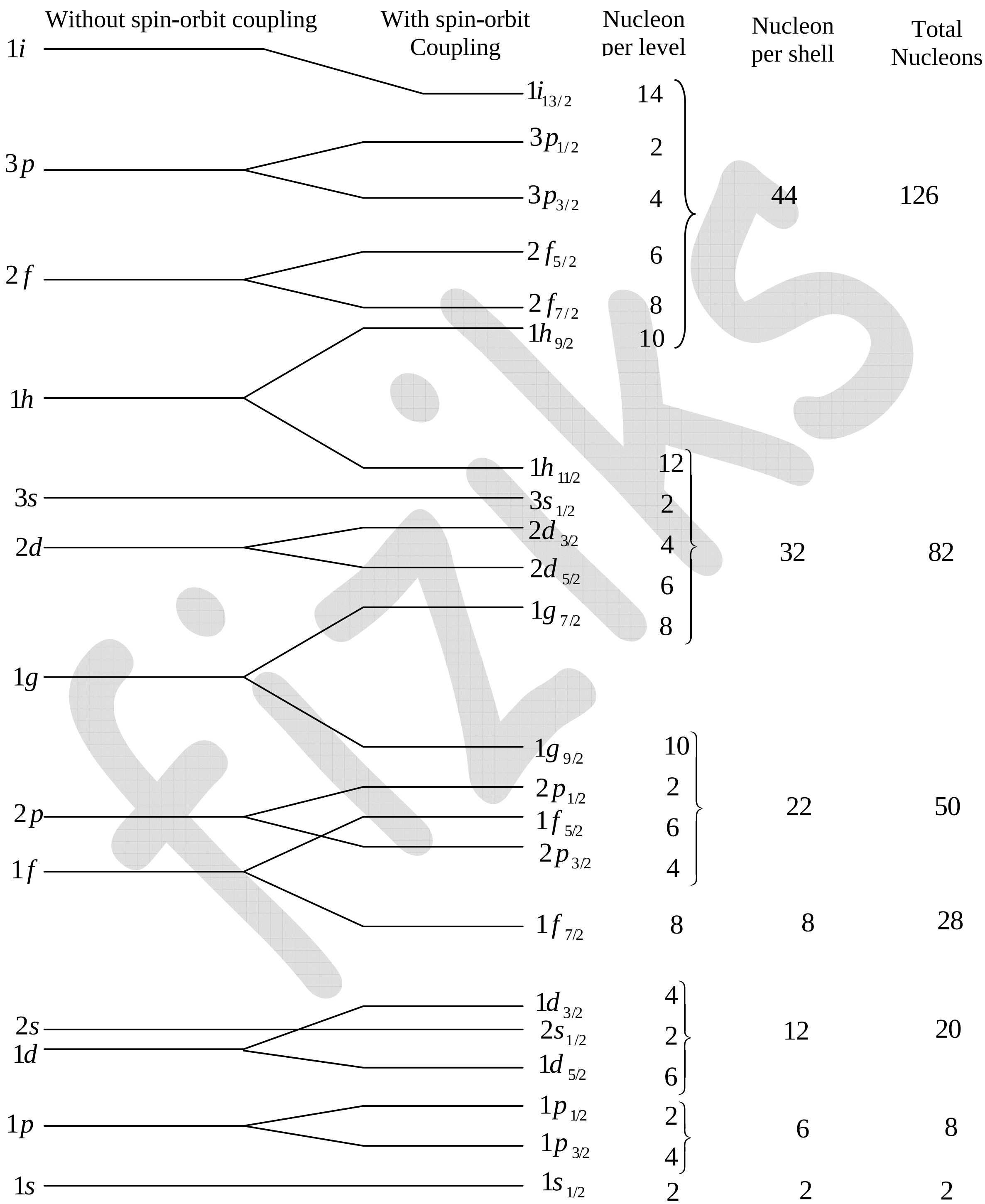
(iii) Since the nuclear forces are not fully known, the potential field cannot be calculated and assumed that it is fairly constant within the nucleus and changes rapidly near the edges. A reasonable guess on the basis of the nuclear density curves is a square well with rounded corners. Schrödinger equation for a particle in a potential well of this kind is solved and it is found that stationary states of the system occur that are characterized by quantum numbers n (total quantum number), l (angular momentum quantum number that can take values from 0 to $n-1$) and m_1 (magnetic quantum number that can take values from $-$l to $+$l).

However unlike atomic shells, here the stationary states having lower 1 quantum number have higher energy. $l = 0,1,2,3,4,5$ etc are represented by alphabets s, p, d, f, g, h respectively, also the quantum number m_l can take $2l+1$ values. The energy levels that come from such calculation do not agree with the observed sequence of magic numbers. Something essential is missing from the picture.

The problem is solved by incorporating spin orbit coupling whose magnitude is such that the consequent splitting of energy levels into sub levels is many times longer than analogous splitting of atomic energy levels.

The spin orbit coupling can be either LS coupling jj coupling. LS coupling hold for very lighter nuclei only. It is the jj coupling which holds for great majority of nuclei. In jj - coupling $\bar{s}_i$ and $\bar{l}_i$ of each particle are first coupled to form $\bar{j}_i$ for the particle of magnitude $\sqrt{j(j+1)}\hbar$ where $j = l \pm s = l \pm \frac{1}{2}$. The various $\bar{j}_i$, then couple together to form the total angular momentum $\bar{J}$.

When appropriate strength is assumed for the spin orbit interaction, the energy levels of either class of nucleon fall into the sequence as shown in figure below.



The levels are designated by a prefix equal to total quantum number n, a letter that indicates l for each particle in that level according to the usual pattern (s, p, d, f, g corresponding to $l = 0, 1, 2, 3, 4$ respectively) and a subscript equal to j. The spin orbit interaction splits each state of a given j into $2j+1$ sub states which can accommodate $2j+1$ nucleons.

Large energy gaps appear in the spacing of the levels at intervals that are consistent with the notion of separate shells. The number of available nuclear states in each nuclear shell is in the ascending order of energy $2, 6, 12, 8, 22, 32$ and 44. Hence shell are filled when there are $2, 8, 20$, $28, 50, 82$ and 126 neutron or protons in a nucleus giving as a result stable nucleus which are relatively more abundant in nature. Thus shell model accounts for the phenomenon of magic numbers.