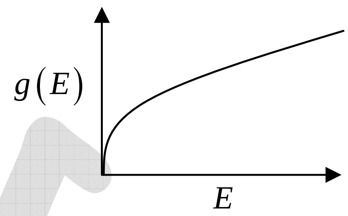
(b) Density of State in Three Dimension

$$g(E)dE = \frac{1}{h^3}\int\int dx.dp_x \int\int dy.dp_y \int\int dz.dp_z \Rightarrow g(E)dE = \frac{V}{h^3}\iiint dp_x dp_y dp_z$$

where V is volume of container

$$\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m} = E \Rightarrow p_x^2 + p_y^2 + p_z^2 = \left(\sqrt{2mE}\right)^2$$



which is equation of sphere with coordinate p_x, p_y, p_z.

So, $\iiint dp_x dp_y dp_z$ is volume of sphere with radius $\sqrt{2mE}$

$g(E)dE = \frac{V}{h^3}\iiint dp_x dp_y dp_z$ is equivalent to $\frac{V}{h^3} \times \phi(E)$

where $\phi(E)$ volume of spherical shell between radius $\sqrt{2m(E+dE)}$ and $\sqrt{2mE}$

$$\phi(E) = \frac{4\pi}{3}\left((2m(E+dE))^{3/2} - (2mE)^{3/2}\right) = \frac{4\pi}{3}(2mE)^{3/2}\left(\left(1+\frac{dE}{E}\right)^{3/2} - 1\right)$$

Using Taylor expansion then $\frac{4\pi}{3}(2mE)^{3/2}\left(\left(1+\frac{3}{2}\frac{dE}{E}\right) - 1\right) \Rightarrow 2\pi(2m)^{3/2} E^{3/2} dE$

$g(E)dE = \frac{V}{h^3}\iiint dp_x dp_y dp_z$ is equivalent to $\frac{V}{h^3} \times 2\pi(2m)^{3/2} E^{3/2} dE$

$g(E)dE$ in three dimension $g(E)dE = 2\pi V\left(\frac{2m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$ where V is volume of three dimensional space.