(d) Kinetic Interpretation of Temperature

According to the assumption of kinetic theory of gases, there is only translation motion of the molecule and there is not any potential acting between them, so

Average energy $\langle E\rangle$ of gases are equivalent to average translational energy of a molecule i.e., $\langle E\rangle = \frac{1}{2}m\langle v^2\rangle$

Pressure, $PV = \frac{1}{3}mN\langle v^2\rangle = \frac{2}{3}N\left(\frac{1}{2}m\langle v^2\rangle\right) = \frac{2}{3}N\langle E\rangle$

$$PV = \frac{2}{3}N\langle E\rangle \text{, where } PV = nRT \Rightarrow PV = \frac{N}{N_A}RT \Rightarrow PV = Nk_BT$$

$$PV = \frac{2}{3}N\langle E\rangle \text{ put value of } PV = Nk_BT$$

$\langle E\rangle = \frac{3}{2}k_BT$, where k_B is Boltzmann constant $k_B = 1.38\times 10^{23}\, J/K$

So, average kinetic energy is given by $\langle E\rangle = \frac{3}{2}k_BT$, where T is absolute temperature.

Internal energy of gases with degree of freedom

Number of independent coordinates required to specify its position correctly

$$f = 3N - k$$

Where N is number of particles and k is Total number of constraints. For a system in equilibrium at temperature T, the total energy is equally partitioned among the different degrees of freedom and Energy associated with each degree of freedom is $\frac{1}{2}k_BT$. If there is f degree of freedom then internal energy of system molecule is $U = \frac{fk_BT}{2}$

If U is internal energy of Ideal gas then specific heat at constant volume is defined as $C_V = \left(\frac{\partial U}{\partial T}\right)_V = f.k_B$

Example: Mono atomic gas in three dimensional volume:

$$E_{trans} = \frac{1}{2}m\left[\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right]$$

$$f = 3\times 1 - 0 = 3 \Rightarrow f = 3$$

$$U = \frac{3}{2}k_B T \quad C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{3}{2}R,\ C_P = \frac{5}{2}R\ ,\gamma = \frac{5}{3} = 1.66$$

Example: Diatomic molecule at low temperature:

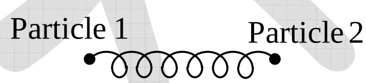
At low temperature Diatomic molecule can be under stood by a model in which two points mass are connected by a mass less rod .so distance between both of particle is constant, so $k = 1$.

So, degree of freedom is $f = 3\times 2 - 1 = 5$

$$U = \frac{f}{2}k_B T \Rightarrow U = \frac{5}{2}k_B T \quad C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{5}{2}R,\ C_P = \frac{7}{2}R\ ,\gamma = \frac{7}{3} = 1.4$$

Example: Diatomic molecule at high temperature:

At high temperature Diatomic molecule can be under stood by a model in which two point mass are connected by a mass less spring. Due to spring one extra mode of degree of freedom (one due to potential energy) will excited hence



So degree of freedom is $f = f(K.E) + f(P.E) = (3\times 2 - 0) + 1 = 7$

$$U = \frac{f}{2}k_B T \Rightarrow U = \frac{7}{2}k_B T \quad C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{7}{2}R,\ C_P = \frac{9}{2}R\ ,\gamma = \frac{9}{7} = 1.28$$

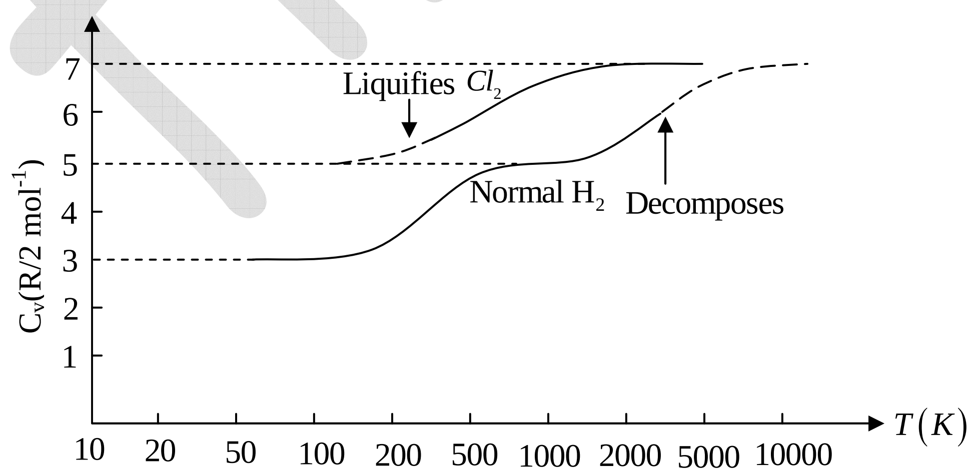


Figure 1: Variation of C_V with temperature $T(K)$